A Fuzzy Cognitive Map Based on the Random Neural Model

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Abstract. A fuzzy cognitive map is a graphical means of representing arbitrarily complex models of interrelations between concepts. The purpose of this paper is to describe a fuzzy cognitive map based on the random neural network model called the random fuzzy cognitive map, and to illustrate its application in the modeling of process. This model is based on the probability of activation of the neurons/concepts in the network. Our model carries out inferences via numerical calculation instead of symbolic deduction.

1 Introduction

Fuzzy Cognitive Maps (FCM) was proposed by Kosko to represent the causal relationship between concepts and analyze inference patterns [5], [6]. FCM is a hybrid method that lies in some sense between fuzzy systems and neural networks. So FCM represent knowledge in a symbolic manner and relates states, processes, policies, events, values and inputs in an analogous manner. FCM have gained considerable research interest and have been applied to many areas [3], [6], [7], [8], [9]. The Random Neural Network (RNN) has been proposed by Gelenbe in 1989 [4]. This model calculates the probability of activation of the neurons in the network. The RNN has been used to solve optimization and pattern recognition problems [1], [2]. The problem addressed in this paper concerns the proposition of a FCM, using the RNN. We describe the Random Fuzzy Cognitive Map (RFCM) and illustrate its application in the modeling of process. We shall use each neuron to model a concept. In our model, each concept is defined by a probability of activation, the relationships between the concepts are defined by positive or negative interrelation probabilities, and the procedure of how the cause takes effect is modeled by a dynamic system. This work is organized as follows, in section 2 the theoretical bases of the RNN and of the FCM are presented. Section 3 presents our RFCM. In section 4, we present applications. Remarks concerning conclusions are provided in section 5.

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2 Theoretical Aspects

2.1. The Random Neural Network Model

The RNN model consists of a network of *n* neurons in which positive and negative signals circulate [4]. Each neuron *i* accumulates signals as they arrive, and can fire according to an exponential distribution of constant rate r(i) if its total signal count at a given instant of time is positive. Positive and negative signals have different roles in the network. A negative signal reduces by 1 the potential of the neuron to which it arrives or has no effect on the signal potential if it is already zero; while an arriving positive signal adds 1 to the neuron potential. Signals can either arrive to a neuron from the outside of the network or from other neurons. A signal which leaves neuron *i* heads for neuron *j* with probability $p^+(i,j)$ as a positive signal (excitation), or as negative signal with probability $p^-(i,j)$ (inhibition), or it departs from the network with probability d(i). Positive signals arrive to the i^{th} neuron according to a Poisson process of rate $\Lambda(i)$. Negative signals arrive to the i^{th} neuron according to a Poisson process of rate $\lambda(i)$. The main property of this model is the excitation probability of a neuron *i*, q(i), which satisfy the non-linear equation:

$$q(i) = \lambda^+(i)/(r(i)+\lambda^-(i)) \tag{1}$$

where, $\lambda^+(i) = \sum^n_{j=1} q(j)r(j)p^+(j,i)+\Lambda(i)$

$$\lambda^-(i) = \sum^n_{j=1} q(j)r(j)p^-(j,i)+\lambda(i)$$

The synaptic weights for positive ($w^+(i,j)$) and negative ($w^-(i,j)$) signals are: $w^+(i,j) = r(i)p^+(i,j)$ and $w^-(i,j) = r(i)p^-(i,j)$. Finally, $r(i) = \sum^n_{j=1} [w^+(i,j) + w^-(i,j)]$

2.2 Fuzzy Cognitive Maps

FCMs combine the robust properties of fuzzy logic and neural networks. FCM was proposed by Kosko [6], [7]. A FCM describes the behavior of a system in terms of concepts, each concept represents a state or a characteristic of the system. Particularly, a FCM is a fuzzy signed oriented graph with feedback that model the worlds as a collection of concepts and causal relations between concepts. Variable concepts are represented by nodes. The graph's edges are the casual influences between the concepts. The causal relationships are expressed by either positive or negative signs and different weights. The value of a node reflects the degree to which the concept is active in the system at a particular time. This value is a function of the sum of all incoming edges multiplied and the value of the originating concept at the immediately preceding state. In general, a FCM functions like associative neural networks. A FCM describes a system in a one-layer network which is used in unsupervised mode, whose neurons are assigned concept meanings and the interconnection weights represent relationships between these concepts. The fuzzy indicates that FCMs are often comprised of concepts that can be represented as fuzzy sets and the causal relations between the concepts can be fuzzy implications, conditional probabilities, etc. A directed edge E_{ij} from concept C_i to concept C_j measures how much C_i causes C_j. In general, the edges E_{ij} can take values in the fuzzy causal interval [-1, 1] allowing degrees of causality to be represented: a) $E_{jk}>0$ indicates direct (positive) causality between concepts C_j and C_k; b) $E_{jk}<0$ indicates inverse (negative) causality between concepts C_j and C_k; c) $E_{jk}=0$ indicates no relationship between C_j and C_k.

In FCM nomenclature, model implications are revealed by clamping variables and using an iterative vector-matrix multiplication procedure to assess the effects of these perturbations on the state of a model. A model implication converges to a global stability. During the inference process, the sequence of patterns reveals the inference model. The development of a FCM often occurs within an expert group. Each expert provides its individual FCM matrix, which is then synthesized into a group FCM matrix. The group matrix (E^G) could be computed as:

$$E_{ji}^{G} = \max_{t}\left\{E_{ji}^{t}\right\} \tag{2}$$

Where E_{ji}^{t} is the opinion of the expert *t* about the relationship among C_j and C_i.

3 Our Random Fuzzy Cognitive Maps (RFCM)

Our RFCM improves the conventional FCM by quantifying the probability of activation of the concepts and introducing a nonlinear dynamic function to the inference process. The value of W_{ij} indicates how strongly concept C_i influences concept C_j. $W^+_{ij}>0$ and $W^-_{ij}=0$ if the relationship between the concepts C_i and C_j is direct, $W^-_{ij}>0$ and $W^+_{ij}=0$ if the relationship is inverse, or $W^+_{ij}=W^-_{ij}=0$ if doesn't exist a relationship among them. To calculate the state of a neuron on the RFCM (the probability of activation of a given concept C_j), the following expression is used:

$$q(j) = \min\left\{\lambda^{+}(j), \max\left\{r(j), \lambda^{-}(j)\right\}\right\} \tag{3}$$

where
$$\lambda^{+}(j) = \max_{i=1,n}\left\{\min\left\{q(i), W^{+}(i,j)\right\}\right\}$$
$$\lambda^{-}(j) = \max_{i=1,n}\left\{\min\left\{q(i), W^{-}(i,j)\right\}\right\}$$

Such as, $\Lambda(j)=\lambda(i)=0$. In addition, the fire rate is:

$$r(j) = \max_{i=1,n}\left\{W^{+}(i,j), W^{-}(i,j)\right\} \tag{4}$$

The general procedure of the RFCM is the following:

1. Design the configuration of the FCM. Experts determine the concepts and causality.
2. Initialize the number of neurons (concepts).
3. Call the Learning phase
4. Call the Simulation phase.

3.1 The Learning Procedure

In this phase we must define the weights. The weights are defined and/or update as:

- *Based on expert's opinion:* each expert defines its FCM and the global FCM is determined according to the equation (2). The next algorithm determines the weight from a group of experts: a) If i≠j and if $E_{ij}>0$, then $W_{ij}^{+} = \max_{t=1,NE}\left\{E_{ij}^{t}\right\}$ and $W_{ij}^{-} = 0$, b) If i≠j and if $E_{ij}<0$, then $W_{ij}^{-} = \max_{t=1,NE}\left\{E_{ij}^{t}\right\}$ and $W_{ij}^{+} = 0$; c) If i=j or if $E_{ij}=0$, then $W_{ij}^{+} = W_{ij}^{-} = 0$. The causal relationship (E_{ij}) is caught from each expert using the next set: E= {no relationship=>0, Slight=>0.2, Low=>0.4, Somehow=>0.6, Much=> 0.8, Direct=> 1}.
- *Based on measured data:* In this case we have a set of measures about the system. This information is the input pattern: $M=\{D_1, \ldots, D_m\} = \{[d_1^1, d_1^2, \ldots, d_1^n], \ldots, [d_m^1, d_m^2, \ldots, d_m^n]\}$, where d_j^t is the value of the concept C_j measured at time *t*. In this case, our learning algorithm follows the next mechanism:

$$W_{ji}^{t} = W_{ji}^{t-1} + \eta\left(\Delta d_{j}^{t} \Delta d_{i}^{t} \big/ \Delta^{+} d_{i}^{t} \Delta^{+} d_{j}^{t} \right)$$

where $\Delta d_j^t = d_j^t - d_j^{t-1}$ $\qquad \Delta d_i^t = d_i^t - d_i^{t-1}$

$\Delta^+ d_j^t = d_j^t + d_j^{t-1}$ $\qquad \Delta^+ d_i^t = d_i^t + d_i^{t-1}$

η is the learning rate.

3.2 The Simulation Phase

Once constructed the RFCM of a specific system, we can perform qualitative simulations of the system. The RFCM can be used like an auto-associative memory. In this way, when we present a pattern to the network, the network will iterate until generate an output close to the information keeps. This phase consists on the iteration of the system until the system convergence. The input is an initial state $S_0= \{s_{1,\ldots}, s_n\}$, such as $q^0(1)=s_1, \ldots, q^0(n)=s_1$ and $s_i \in [0, 1]$ (set of initial values of the concepts ($S_0=Q^0$)). The output $Q^m=\{q^m(1), \ldots, q^m(n)\}$is the prediction of the RFCM such as *m* is the number of the iteration when the system converge. The algorithm of this phase is:

1. Read input state Q^0

2. Calculate the fire rate r(i)
3. Until system convergence
   3.1 Calculate q(i)

4 Experiments

In this section we illustrate the RFCM application. A discrete time simulation is performed by iteratively applying the equation (4) to the state vector of the graph. At the beginning, we must define an initial vector of concept states, and the simulation halts if an equilibrium state is reached. In our experiment, we discuss a simple model to determine the risk of a crisis in a country. The operative concepts are: a) Foreign inversion (C_1): the presence of a strong foreign inversion; b) Employ rate (C_2): The level of employ on the country; c) Laws (C_3): the presence or absence of laws; d) Social problems (C_4): the presence or absence of social conflict on the country; e) Government stability (C_5): a good relationship between the congress, the president, etc. The edge connection matrix (E) for this map is given in table 1.

Table 1. The edge connection matrix for the first experiment.

	C_1	C_2	C_3	C_4	C_5
C_1	0	0.8	0	0	0
C_2	0	0	0	-06	0.8
C_3	0.4	0	0	-0.8	0
C_4	0	0	0	0	-0.8
C_5	0.6	0	0	0	0

The table 2 presents the results for different initial states. Clamping two antithetical concepts allows to test the implications of one or more competing concepts. To illustrate, we begin by clamping C_1 and C_4 (S_0=(1 0 0 1 0)) – a strong foreign inversion can generate more employment. Despite of the foreign inversion, we have an unstable government due to the social problems (the system reaches an equilibrium state of (1 1 0 0 1)). With S_0=(1 0 1 1 0) foreign inversion and social problems remain clamped, but we also clamp the ability to have a good law system. The system reaches an equilibrium state of 1 1 1 0 0 – A peaceful country at the social level but one unstable government. Next, we test for S_0=(1 1 1 1 0). In our model, the inference process is: S_1 = (0.8 .0.6 0 1 0.8), S_2 = (0.8 .0.6 0 0.8 0.4), and S_3 = (0.8 .0.6 0.8 0.6 0). In this example, we could take advantage of the ability to study the inference process during execution of the simulation. This example suggests the social problem is the main factor to have an unstable government. Obviously, our goal in analyzing this model was not to determine policy choices for a country. Rather, we tried to illustrate the advantages of the RFCM to this sort of analysis. Our results indicate that RFCMs quickly come to an equilibrium regardless of the complexity of the model.

Table 2. The results for the first experiment.

Input	Kosko FCM	RFCM	Iteration
1 0 0 1 0	1 1 0 1 0	0.8 0.6 0.2 0.2 0.8	1
	1 1 0 0 1		2
1 0 1 1 0	1 1 1 1 0	0.8 .0.6 0.8 0 0	1
	1 1 1 0 0		2
1 1 1 1 0	1 1 1 1 0	0.8 .0.6 0 1 0.8	1
		0.8 .0.6 0 0.8 0.4	2
		0.8 .0.6 0.8 0.6 0	3

5 Conclusions

In this paper, we have proposed a FCM based on the Random Neural Model, the RFCM. We have shown that this model can efficiently work as associative memory. Our RFCM exhibit a number of desirable properties that make it attractive: a) Provide qualitative information about the inferences in complex social dynamic models; b) Can represent an unlimited number of reciprocal relationships; c) Facility the modeling of dynamic, time-evolving phenomena and process. Another important characteristic is its simplicity, the result of each RFCM's cycles is computed from the equation (4). Further on, we will study the utilization of the RFCM in modeling the behavior of distributed systems and dynamic systems. In addition, we will test our unsupervised learning approach.

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