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Master Thesis

"Different approaches with fuzzy cognitive maps in the modelling of granular time series"

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1. Introduction

The relationship between data and time and its complexities is a subject that has had a great relevance in econometrics. In general, a vast amount of the data is ordered by time, which actually is the definition for a time series, and therefore, has properties that require a specific approach. This kind of data is crucial for decision-making in all types of industries, since time is an essential factor when taking decisions. This is one of the reasons on why it is a problem that has been addressed through different methodologies to predict its behavior. Among these, the most popular perspectives might be classified into classical statistics and machine learning. In this study a hybrid methodology will be considered.

Something in common with classical statistical models, such an ARIMA, AR, MA, ARMA, SARIMA…, and machine learning models, such as neural networks, is the fact that the forecast will be done point by point, in terms of the granularity of the time series. In this way, the accuracy is measured by the differences of the predicted values and the real values, where all the forecasted points have the same importance. But there might be values in the time series that might have a larger importance, such as minimum, maximum, average, median, among others.

A question on whether these traditional models are the most adequate models to be used in situations in which the focus is on the accuracy of the beforementioned parameters is actually raised . For instance, a bank could be mostly interested on the minimum interest rate for the following year, or the average probability of the default for the next three months. The weather forecast could be interested in the minimum temperatures, which could affect the timing of the planting for agriculture. Another example could be the maximum number of products that will be sold in the next Black Friday, or the maximum number of users that the selling web needs to attend, which would affect the organization of a company in terms of production and organization.

All these examples point out that, sometimes, the focus of forecasting a time series for certain problems is rather on fewer values with high accuracy rather than forecasting all the values. This is the motivation behind of the current study.

Based on the article [1], the approach to best deal with these kinds of problems will be done by the granularization of the time series and the posterior analysis with fuzzy cognitive maps ("**FCM**"). The FCM are a type of artificial neural network (ANN) [1]. They use nodes to represent concepts, which in our case will be the second-phase granules, to display causal relationships between them.

This study will extend the methodology set in [1] by:

- Applying the methodology with different number of clusters to three different case studies:
  - Trend time series
  - Stationary time series
  - Seasonal time series

- Optimizing the FCM with a genetic optimization algorithm [1] and with an additional non-linear solver (Internal Point Optimizer).
- Including a new function to measure the strength of membership between the second-phase granules with the first-phase granules.
- Including a new approach to forecast the values of the first-phase granules.

The ultimate goal is to analysis the behavior of our general approach for different variations of the main methodology for time series with different behaviour and different properties, and analyze the best fit for each one.

To do so, this study is organized as follows: Section 2 contains the theoretical background and the details of all the steps followed in the process. Section 3 contains the contributions and new findings. Section 4 contains the results of the model with different time series, and finally, Section 5 contains the conclusions.

2. Theoretical background

Section 2 explains the theoretical methodology developed by the original article [1]. The contributions based on modifications of equations of the original article or based on the incorporation of new methodologies can be found in Section 3.

The modelling starts with a raw time series, which is the data with no transformation. The first novelty of this model is the granularization. This means that rather focusing on each point of the raw time series, the focus is on intervals of the time series. The intervals must be of equal length. The selection of the length of the interval might depend on the needs of the forecast or expert judgment, which is ultimately related with the nature of the time serie being analyzed.

Once the intervals are defined, each interval will be summarized in three values (minimum, maximum and median) through the Principle of Justifiable Granularity. The raw time series gets transformed into the so called first-phase granules, or granules. There will be as many first-phase granules as total splits of same length intervals of the time series.

These first-phase granules will be clustered through the fuzzy C-means into the so called second-phase granules, which are the centres of the clusters. The second-phase granules are also the concepts of the FCM. These concepts are assigned to a linguistic term depending on their values, such as “low” or “high”.

The strength between each second-phase granule will be computed through the techniques to measure the strength of the membership and these values will be optimized with the FCM.

In this section, the following ideas will be covered:

1. First-phase granulation of the time series: Used to show the basic logic of the granulation (Shown in Figure 1 as step 1).
2. Principle of Justifiable Granularity (PJG): Used to compute the triangular fuzzy numbers, which are the first-phase granules (Shown in Figure 1 as step 2).
3. Fuzzy C-Means clustering: Used to determine the clusters that will be the concepts of the FCM, which are named as the second-phase granules (Shown in Figure 1 as steps 3 and 4).
4. Conceptual description of the first-phase granular time series by the second-phase granules: Used to measure the strength of each first-phase granule with each second-phase granule.
5. Fuzzy cognitive maps (FCM): Used to compute the weights between the clusters or the second-phase granules.

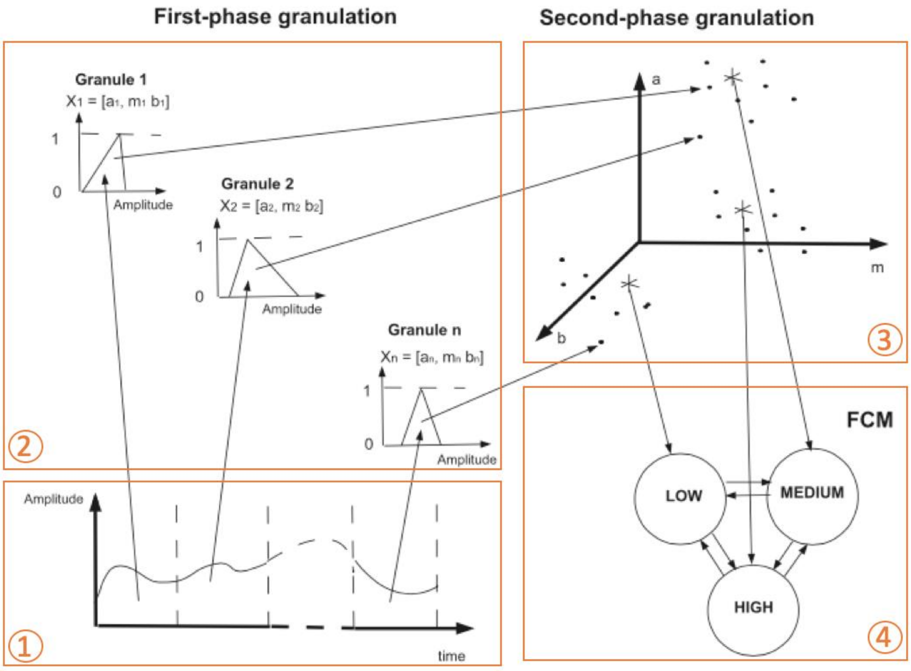


Figure 1 Model summary. Figure obtained from original article [1]

2.1. First-phase granulation of the time series

Let $\{X\} = \{x(0), x(1), \dots, x(n)\}, t \in [0, n]$ be the time series. For future terminology, the first-order differences of the series will be denoted as $\{\partial X\} = \{\Delta x(0), \Delta\, x(1), \dots, \Delta x(n)\}, t \in [0, n]$ where $\Delta x(i) = x(i) - x(i-1)$. Also let the number of intervals be N, which is directly linked with the length of each interval (w), where $N = \frac{n}{w}$ and $w \geq 2$. After the process of defining the length and number of intervals we have created a new interval-based time scale $k \in [0, N]$.

As previously mentioned, having the same length is a constraint on creating these intervals, so depending on the w, we might need to remove data at the beginning of the time serie to make all the intervals of equal length. Logically, this will depend on the length of the time series. If we have a sufficient volume of data available, it will be viable; otherwise, we will need to adapt the length of the intervals to the volume of data we have.

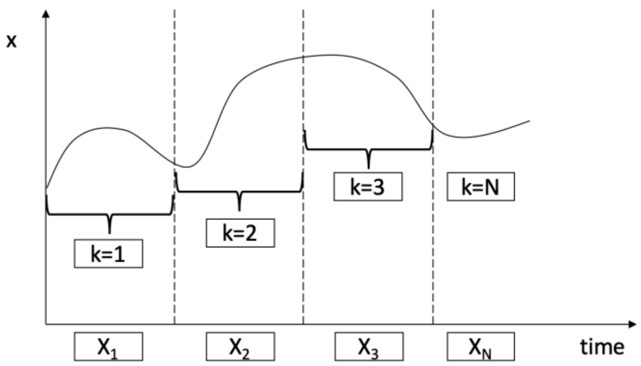


Figure 2 Time series correspondence with phirst-phase granules with N=4

Let us denote $G^1(\cdot)$ as the first-phase granulation process, which will produce a granule for each k^{th} interval, as shown in Figure 2. The first-phase granulation process is done by the Principle of Justifiable Granularity, defined in [1].

2.2. Principle of justifiable granularity

The definition of granule can be done in different ways [7, 8] but in this study, we stick to the definition done by [1] and so, the raw granules of the time series will be explained as a triangular fuzzy number.

Each raw granule´s values x are real values that belong to the interval $[x_{min}, x_{max}]$, where x_{min} and x_{max} refer to the maximum and minimum value of the granule. As a quick note, these values are not related with the time position, which means that they could be found at any time in the time interval of the raw granule.

The triangular fuzzy number X is denoted as $X(x; m, a, b)$ where m is the median value [3], a is a value that belongs to the interval $[x_{min}, m]$ and b is a value that belongs to the interval $[m, x_{max}]$, this is $x_{min} \leq a < m < b \leq x_{max}$.

The values a and b are values that will be optimized by discomposing $X(x; m, a, b)$ into two linear functions. All the process of computing the optimal values of a and b will be done with formulas that refer to the left-hand side of the function and formulas that refer to the right-hand side of the function. In this case, Equation 1 refers to the left-hand side of the function and Equation 2 refers to the right-hand side of the equation:

$$X(x; m, a) = \frac{x - a}{m - a} \tag{1}$$

$$X(x; m, b) = \frac{x - b}{m - b} \tag{2}$$

This is the first step to develop the first-phase granule. Two measures that are contradictory need to be addressed to optimize the values of the granule, which are the specificity and the coverage. The coverage measures the amount of points that are kept in the interval $a \leq m \leq b$. The Equation 3 refers to the left-hand side of the equation and Equation 4 refers to the right-hand side of the equation:

$$cov([a, m]) = \sum_{x \in [a,m]} X(x; m, a) \tag{3}$$

$$cov([m, b]) = \sum_{x \in [m,b]} X(x; m, b) \tag{4}$$

The amount of information contained in a first-phase granule is measured by the specificity. It also evaluates the degree of belonging of the points to the granule. The Equation 5 refers to the left-hand side of the equation and Equation 6 refers to the right-hand side of the equation:

$$sp([a, m]) = 1 - \frac{0.5 \cdot |m - a|}{|m - x_{min}|} \tag{5}$$

$$sp([m,b]) = 1 - \frac{0.5 \cdot |b-m|}{|x_{max}-m|} \tag{6}$$

The higher the value of the specificity the more useful the information is. If we had $m = b$, then we would keep the full interval, and thus, the specificity would be equal to 1, and in this situation, the interval length would be 0.

Finally, to optimize the coverage and specificity, the performance index Q will be optimized. The Equation 7 refers to the left-hand side of the equation, and Equation 8 refers to the right-hand side of the equation:

$$Q(a) = cov([a,m]) \cdot sp([a,m]) \tag{7}$$

$$Q(b) = cov([m,b]) \cdot sp([m,b]) \tag{8}$$

And finally, the values of a and b are selected by maximizing the previous formulas according with the Equation 9 for the left-hand side of the equation and Equation 10 for the right-hand side of the equation:

$$a_{opt} = argmax_a Q(a) \tag{9}$$

$$b_{opt} = argmax_b Q(b) \tag{10}$$

As it can be noted, for the entire process, the optimization of a and b is done separately, so the value of a holds no relation with the value of b. This process has been carried on by computing a grid of values and using the ones that maximize Equation 9 and Equation 10.

All this process leads to the first-phase granules, represented in Figure 3, where the green doted lines represent the raw granules, and the black straight lines represent the optimized granule by the values of a and b.

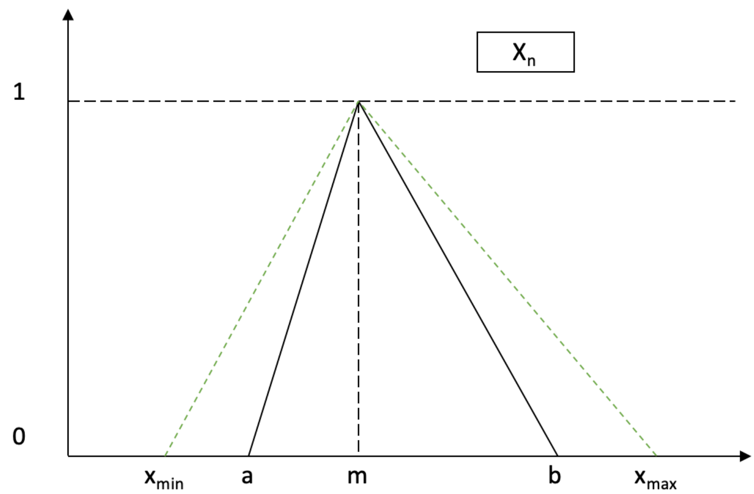


Figure 3 Representation of result of Principle of Justifiable Granularity

Once the first phase granules are obtained, the Fuzzy C-Means will be used to compute the second-phase granules, which will be later used as the concepts of the FCM. Ultimately, the second-phase granules will be the key elements to build the FCM, and the clustering of the first-phase granules is done in terms of their similarity with the beforementioned second-phase granules (Thus, it is good to clarify that the concepts, clusters and granules of the second phase are the same).

2.3. Fuzzy C-Means clustering

The Fuzzy C-Means algorithm [4] will be the tool used to compute the second-phase granules. This is done by an iterative process that seeks to minimize the objective function of Equation 11 in terms of the underlying errors between the first-phase granules and the second-phase granules.

Let us assume $x_i \in X^d$ is the first-phase granule in d – dimensional space, and a_j be a center of the cluster A_j formed in the d – dimensional space. This is, a_j are the second-phase granules. The Equation 11 will be the objective function to minimize to generate the second-phase granules and cluster the first-phase granules into them:

$$J = \sum_{i}^{N} \sum_{j}^{c} u_{ij}^{m} \left\| x_i - a_j \right\|^2, \tag{11}$$

Where:

- $m \geq 1$,
- N is the cardinality of the dataset X^d (number of intervals),
- c is the number of clusters (number of second-phase granules),
- u_{ij} is the degree of membership of x_i in the cluster A_j,
- $\|.\|$norm measuring the similarity between the data and the center of the cluster a_j. In this scenario, the Euclidean norm will be used.

This process is carried on until a certain number the iterations is reached, or an error criterion is met. In every iteration k^{th}, the membership function is updated with the Equation 12:

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{\left\| x_i - a_j \right\|}{\left\| x_i - a_k \right\|} \right)^{\frac{2}{m-1}}} \tag{12}$$

Where the position of the cluster is updated with the Equation 13:

$$a_j = \frac{\sum_{i}^{N} u_{ij}^{m} x_i}{\sum_{i}^{N} u_{ij}^{m}} \tag{13}$$

As stated before, the iteration process is finished when a certain number of iterations is reached, or the error is minimized according with the Equation 14:

$$max_{ij}\left(\left| u_{ij}^{k+1} - u_{ij}^{k} \right| \right) < \varepsilon \tag{14}$$

Where $\varepsilon \in [0,1]$.

As a result of this iterative process, the second-phase granules are generated. On the left-hand side of Figure 4, there is an alternative representation of the first-phase granules with three axes (a, b and m). On the right-hand side, the red dots would represent the second-phase granules (centers) among the first-phase granules.

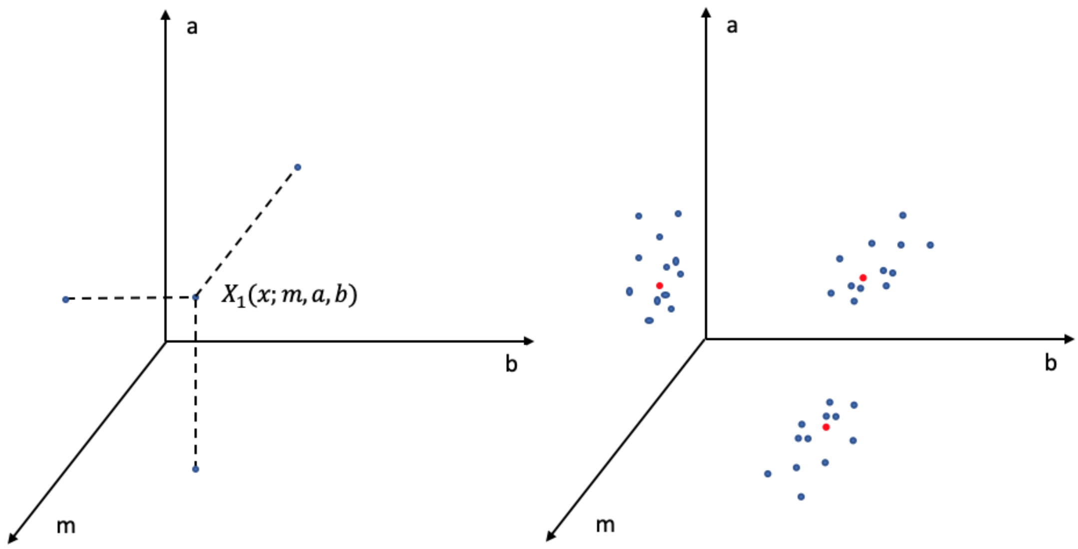


Figure 4 Representation of granules in 3D and result of clustering

2.4. Conceptual description of the First-Phase Granular time series by the Second-Phase Granules

Once the first-phase granules and the second-phase granules are created, the link between both types of granules needs to be constructed, this is explaining $\{X_k\}$ in terms of $\{A_j\}$. To this end, a quantitative membership function measuring the strength of the relationship between each combination of first-phase granules and second-phase granules is needed. For the purpose of this study, some functions will be studied to assess this relationship:

1. The first option will be the possibility function, which has been used in [7,8]:

$$Poss(X_k, A_j) = max_x \min(X_k(x), A_j(x)) \tag{15}$$

This calculation method has a situation that needs to be considered, and to this end, the following numeric example will be used. Let us assume we have a first-phase granule, denoted as $X(x; m, a, b)$, with the shape of $X(3.4, 3, 3.9)$ and three second-phase granules, denoted as $A_k(m, a, b)$, $A_1(4, 3.4, 4.6), A_2(8.9, 7.6, 10)$ and $A_3(11.5, 10, 12.5)$. Let us denote min_c as the value of $\min(X_k(x), A_j(x))$, where c is the number of second-phase granule. Then, the result of each combination would be $min_1(3.4, 3, 3.9), min_2(3.4, 3, 3.9)$ and $min_3(3.4, 3, 3.9)$ and by taking the max of each vector we would yield to the same result: 3.9, which means that X has the same strength of relationship with A_1, A_2 and A_3. The right-hand side of the Figure 5 represents the focus on the representation as fuzzy triangles of the first-phase granule X and the second-phase granule A_1, and in the left-hand side of the Figure 5, the representation as fuzzy triangles of the first-phase granule X and the second-phase granules A_1, A_2 and A_3. It can easily be checked that this result does not seem to display the strength of the relationship well since there is one second-phase granule that is overlapping with the first-phase granule, and yet it has the same strength of relationship as the other granules, which are not overlapping with the first-phase granule.

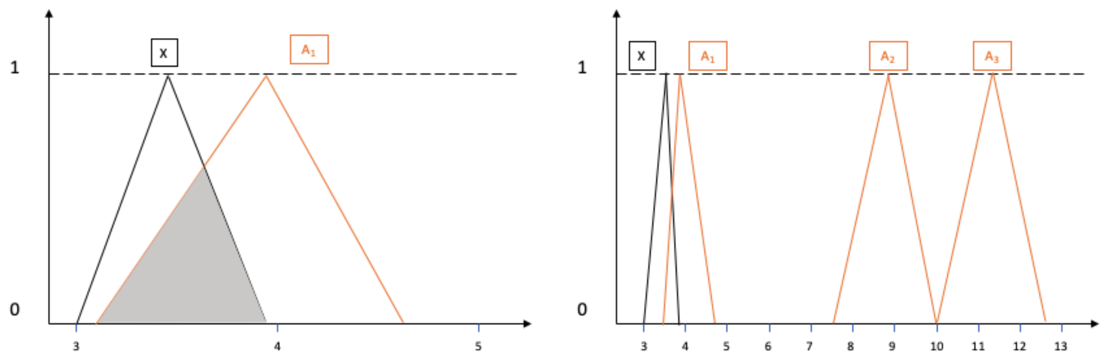


Figure 5 Representation of function 1 to measure the membership strength

When applying this method, the values of $Poss(X_k, A_j)$ will be normalized according to Equation 16:

$$Poss\left(X_k, A_j\right) = \frac{Poss(X_k, A_j)}{\sum_k Poss(X_k, A_j)} \tag{16}$$

Yielding to values that lie between 0 and 1. This is done to avoid extreme values when applying Equation 20 when creating the fuzzy cognitive map.

2. The second option will be the matching function, displayed in the Equation 17.

$$Match(X_k, A_j) = \frac{\sum_k \min(X_k(x), A_j(x))}{\sum_k \max(X_k(x), A_j(x))} \tag{17}$$

By using the numerical example shown before, it can be concluded that the measurement of the strength of the relationship with the clusters is more reliable since $Match_1 = 0.85$, $Match_2 = 0.38$ and $Match_3 = 0.3$. Still, the values for center 2 and center 3 still seem a bit large.

The values of $Match\left(X_k, A_j\right)$ will not be normalized since the denominator is scalating the membership strength value.

For each of these formulas, a vector for each interval containing the strength of the relationship between the granule of the interval and every second-phase granule is created: $< \lambda_1(k), \lambda_2(k), \dots, \lambda_c(k) >$ where c is the number of second-phase granules and k refers to each interval. As a reminder, the number of second-phase granules is determined by expert judgment. For each of these vectors, we select the highest value of λ, which will determine the second-phase granule that has the strongest relationship with the interval, according to Equation 18.

$$j_k = argmax_{j=1,2,\dots,c}(\lambda_j(k)) \tag{18}$$

which points to A_{j_k}, which is the second-phase granule that represents the first-phase granule X_k in terms of the membership function.

After performing this operation for all the intervals, we obtain the sequence of granules that compose the second-phase granular time series: $\{G^2X\} = \{A_{j_1}, A_{j_2}, \dots, A_{j_N}\}$.

Through all this process, the original time series $\{X\}$ has been approximated by the first-phase granular time series $\{G^1X\}$, which has been approximated by the second-phase granular time series $\{G^2X\}$.

2.5. Fuzzy Cognitive Maps

The FCM [5, 6] will be used to stablish the relationships between the second-phase granules, and will be the tool to forecast the second-phase granules.

The FCM is an ordered pair, composed by A, which is the collection of second-phase granules, and W, which is the connection matrix between these granules. The W matrix will have an equal number of rows and columns, that will match the number elements of A, this is, the number of second-phase granules. A visual example of the relationship of the weights and the collection of the second-phase granules is shown in Figure 6, where there are three second-phase granules, and therefore, six weights that connect each of the granules. As it will be shown later, the diagonal of the matrix, this is w_{ij}, for $i = j$, is 0.

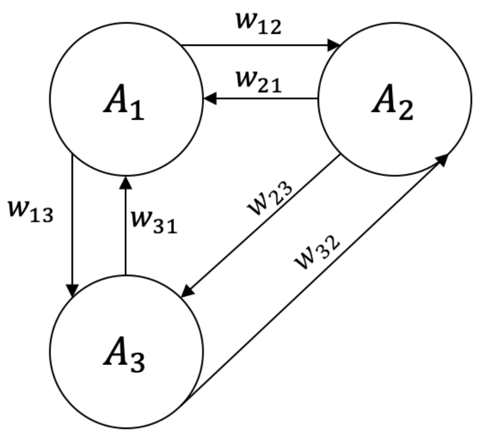


Figure 6 Representation of Fuzzy Cognitive Map

The FCM is applied to one-step ahead forecasting strength of the relationship of each first-phase granule with every second-phase granule. This is shown in Equation 19:

$$\hat{\lambda}_j(t) = f\left(\sum_{i=1, i\neq j}^{c} w_{ij} \cdot \lambda_i(t-1)\right) \tag{19}$$

Where c is the number of centers and $f(z)$ is the selected transformation function, which in this scenario will be the unipolar sigmoid function of Equation 20:

$$f(z) = \frac{1}{1 + e^{-gain \cdot z}} \tag{20}$$

Where the parameter $gain$ determines how quickly the transformation reaches values of 0 and 1. In the current study the value of gain will always be 1.

Note that, each first-phase granule has a different strength of relationship with every second-phase granule, and so, Equation 19 and Equation 20 will be applied for each granule´s λ, which will translate into a number of forecasted λ equal to the number of centers. As a reminder, in this model, the forecast is done by periods, and each period translates into a first-phase granule, the highest value of λ will stablish which is the second-phase granule of the period forecasted.

The optimization of the weights of the FCM has been done through two different optimization processes, which are Interior Point Optimizer (**IPOPT**) [9], proposed in this article as an alternative, and Genetic algorithm. Both of the optimization processes have used historical data, this is, the learning period of the time series. The objective function of both optimization algorithms is to minimize Equation 21:

$$e = \sum_{t=2}^{t} \sum_{j=1}^{M} \varepsilon_j \tag{21}$$

where t is the learning period and M is the cardinality, or number of intervals, of the learning period. Note that the optimization function starts in $t = 2$ since we can not forecast $t = 1$. Also ε_j follows the Equation 22:

$$\varepsilon_j(t+1) = \left|\hat{\lambda}_j(t+1) - \lambda_j(t+1)\right| \tag{22}$$

where $\hat{\lambda}_j$ is the forecasted value of the membership function and λ_j is the actual value of the strength of membership.

After the set of equations defined previously, are aiming to optimize Equation 23:

$$\min_x \sum_{t=2}^{t} \sum_{j=1}^{M} \varepsilon_j \tag{23}$$

$$s.t. \qquad w_{ii} = 0 \; for \; i = 1,2 \dots, c$$

2.6. Forecasting granular time series using Fuzzy Cognitive Maps

Once the FCM has been constructed, we can proceed to do the forecast. The process is as follows:

1. Forecast the strength of the membership function, based on the last membership value of the training partition λ_N, where N is the number of training periods (training first-phase granules). To do so, the Equation 24 will be used, whereas the forecast of the following test values will be done according with Equation 25. We will refer to the membership values of the test as γ to differentiate better from the membership values of the training partition λ.

$$\hat{\gamma}_j(k) = f\left(\sum_{i=1, i\neq j}^{c} w_{ij} \cdot \lambda_{i_N}\right) \tag{24}$$

where N is the number of training periodsand k is assumed to be the testing partition periods.

$$\hat{\gamma}_j(k) = f\left(\sum_{i=1, i\neq j}^{c} w_{ij} \cdot \hat{\gamma}_i(k-1)\right) \tag{25}$$

Note that every time we apply these formulas, a membership value will be obtained for each second-phase granule. The second-phase granule that corresponds with the highest value of the membership function will be the forecasted cluster for the testing period, represented by a first-phase granule.

Once the forecasted first-phase granule is associated with a second-phase granule, the values of $\hat{a}, \hat{b}$ and $\hat{m}$ will be forecasted according with Equation 26, Equation 27 and Equation 28.

$$\hat{a} = \frac{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1) \cdot a_j}{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1)} \tag{26}$$

$$\hat{b} = \frac{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1) \cdot b_j}{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1)} \tag{27}$$

$$\hat{m} = \frac{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1) \cdot m_j}{\sum_{j=1}^{c} \hat{\gamma}_j\,(k+1)} \tag{28}$$

Where a_j, b_j and m_j correspond with the optimized lower and upper bound, and the median of the second-phase granule A_j, respectively.

This approach of forecasting ends up being an average of the values of the centers weighted by the forecasted strength of membership of the first-phase granule that is being forecasted. The rationale behind this approach seems logical and basically the stronger is the membership of the forecasted first-phase granule is with the each second-phase granule, the more weight will have the values of the second-phase granule.

2.7. Evaluation of the forecasting accuracy

To evaluate the forecasting accuracy, two measures are going to be considered:

- Whether the forecast of the second-phase granule is correct, according with the Equation 29.

$$Z = \frac{1}{K}\sum_{k=1}^{K} 1 | A_{jk} \neq \widehat{A_{jk}} \tag{29}$$

- The error in the values of the forecasted membership functions, according with Equation 30 in the testing time horizon $k \in [1, N]$.

$$Error\ type\ 2 = \sum_{k=1}^{N}\left(\sum_{j=1}^{c}\left(\hat{\lambda}_j(k+1) - \lambda_j(k+1)\right)^2\right) \tag{30}$$

- The error of the forecast of the values of $\hat{a}, \hat{b}$ and $\widehat{m}$ is measured by Equation 31, which is the summatory of Equation 32, Equation 33 and Equation 34.

$$Error\ type\ 3 = \varepsilon_a + \varepsilon_b + \varepsilon_m \tag{31}$$

$$\varepsilon_a = \frac{\sum_{k=1}^{N} |\hat{a}(k) - a(k)|}{\sum_{k=1}^{N} |a(k)|} \tag{32}$$

$$\varepsilon_b = \frac{\sum_{k=1}^{N} \left|\hat{b}(k) - b(k)\right|}{\sum_{k=1}^{N} |b(k)|} \tag{33}$$

$$\varepsilon_m = \frac{\sum_{k=1}^{N} |\widehat{m}(k) - m(k)|}{\sum_{k=1}^{N} |m(k)|} \tag{34}$$

$$Error\ type\ 3 = \frac{\sum_{k=1}^{N} \left(|\widehat{m}(k) - m(k)| + \left|\hat{b}(k) - b(k)\right| + |\hat{a}(k) - a(k)|\right)}{\sum_{k=1}^{N} \left(|a(k)| + |b(k)| + |m(k)|\right)}$$ Correccción

3. Contributions and new findings

Section 3 comes as a result of everything learned from the original article [1]. Through the study of the original article, some points were identified as potential room for improvement, which are ultimately explained in this section.

3.1. Fuzzy Cognitive Maps

As stated before, the original article computes the optimization of the FCM with a genetic algorithm. Since the structure of the FCM was thought to be a point to be improved, IPOPT optimization is proposed here as an alternative optimization algorithm to analyze the differences of the activation values between the centers (second-phase granules) of the FCM versus the FCM optimized by the genetic algorithm. IPOPT is a large-scale nonlinear optimization that uses an interior-point method and it is supposed to be efficient for solving problems with continous variables [11], which is the reason of why it is used here.

3.2. Conceptual description of the First-Phase Granular time series by the Second-Phase Granules

The original article proposes two different functions to measure the strength of the membership of the first-phase granules with the second-phase granules (Equation 16 and Equation 17). These functions can be improved, as shown in the comments of Section 2.4: Conceptual description of the First-Phase Granular time series by the Second-Phase Granules. Thus, as an alternative, a third option has been computed in this section, which will consist on computing the normalized Manhattan distance between the first-phase granule and the second-phase granule, as shown in Equation 35.

$$Man(X_k, A_j) = \frac{\left|X_k(x) - A_j(x)\right|}{\sum_k \left|X_k(x) - A_j(x)\right|} \tag{35}$$

By using the numerical example that was shown in Equation 16 and Equation 17, the strength of the membership measured by Formula 18 is $Match_1 = 0.72$, $Match_2 = 0.13$ and $Match_3 = 0.15$. It can be observed that the strength of the relationship with the clusters seems more reliable since the strength is much stronger for the second-phase granule 1 than second-phase granule 2 and second-phase granule 3, which visually holds with Figure 5.

When applying this proposed function, the normalization is done again to avoid extreme values when applying Equation 20 when creating the FCM.

3.3. Forecasting the granular time series using the fuzzy cognitive maps

As an alternative to the proposed approach to forecast the values of the first-phase granule, an additional forecasting approach will be compared with the outputs of the Equation 26, Equation 27 and Equation 28:

$$\hat{a}|A_{jk} = \widehat{A_{jk}} = \frac{a_j + \sum_i^N x_{a_{j_i}}}{N_j + 1} \tag{36}$$

$$\hat{b}|A_{jk} = \widehat{A_{jk}} = \frac{b_j + \sum_i^N x_{bj_i}}{N_j + 1} \tag{37}$$

$$\hat{m}|A_{jk} = \widehat{A_{jk}} = \frac{m_j + \sum_i^N x_{mj_i}}{N_j + 1} \tag{38}$$

Where:

- a_j, b_j and m_j correspond with the optimized lower and upper bound, and the median of the second-phase granule A_j that has the strongest forecasted membership
- N is the cardinality of the dataset X^d (number of first-phase granules),
- x_{aj}, x_{bj} and x_{mj} correspond with the optimized lower and upper bound, and the median of the first-phase granules that have the strongest membership with the second-phase granule A_j

The rationale behind this, is that when having a situation in which the forecasted membership strength is not very different among the different second-phase granules. This is, the forecast is not clear on which second-phase granule should be found in the next period, which is why we end up forecasting a point in the middle of all the clusters, which leads to high errors.

The aim here is to get the average of all the values of the first-phase granules of the cluster (including the values of the corresponding second-phase granule), ignoring the effects of the other clusters. This will yield to forecasting values which only depend on the values of the first-phase granule of the largest membership and thus, it will give a higher importance to correctly forecasting the right second-phase granule.

The values of the Equation 26 and Equation 36, Equation 27 and Equation 37, and finally Equation 28 and Equation 38 are forecasted values for the first-phase granule.

4. Studies with real time series

The objective of Section 4 is to recreate the calculations performed by the original article with three different time series and compare the results with the proposed modifications of Section 3, to test which combination suits better for each type of time series.

4.1. Analysis protocol

Three case studies with different time structure will be studied in the present section:

- Time series with trend.
- Stationary time series (time series with trend differentiated).
- Seasonal time series.

The process of applying the approaches will consist on:

- First-phase granularization.
- Second-phase granularization.
- Comparison of two optimization algorithms with 2, 3, 4 and 5 clusters for the FCM (or second-phase granules) mixed with the different membership functions of Equation 15, Equation 16 and Equation 17.
- Comparison of the forecasting approaches proposed by the original article (Equation 26, Equation 27 and Equation 28) with respect to our proposed approach (Equation 36, Equation 37 and Equation 38).
- Conclusions

The evaluation will be done with a train-test split in which all the case studies will have the same length of test periods, which will be 3, and therefore, 3 test first-phase granules.

The process for all the series will be the same:

1. Apply the Principle of Justifiable Granularity to compute the first-phase granules.
2. Perform the Fuzzy C-means clustering to compute the optimal second-phase granules by applying the Equation 11, Equation 12, Equation 13 and Equation 14. The following references display the decaying errors of the process as iterations go by:
    a. Time series with trend: Figure 13
    b. Stationary time series: Figure 27
    c. Time series with seasonality: Figure 41
3. After this iterative process, the second-phase granules are computed and displayed next by the first-phase granules in the shape of fuzz triangles:
    a. Time series with trend: Figure 14
    b. Stationary time series: Figure 28
    c. Time series with seasonality: Figure 42

    The process of computing the first-phase granules and the second-phase granules will be the same regardless the optimization algorithm and membership calculation used.
4. When computing the FCMs, three different approaches/functions to measure the strength of the membership of the first-phase granules with the second-phase

granulesand two different optimization methods will combine into six different scenarios that will be tested:

a. Scenario 1: Optimization with genetic algorithm and membership measured by Equation 15 and Equation 16.
b. Scenario 2: Optimization with genetic algorithm and membership measured by Equation 17.
c. Scenario 3: Optimization with genetic algorithm and membership measured by Equation 35.
d. Scenario 4: Optimization with IPOPT and membership measured by Equation 15 and Equation 16.
e. Scenario 5: Optimization with IPOPT and membership measured by Equation 17.
f. Scenario 6: Optimization with IPOPT and membership measured by Equation 35.

Since a different number of second-phase granules can be chosen, each process will be tested for 2, 3, 4 and 5 second-phase granules, or clusters. As a reminder, the number of clusters is chosen by expert criteria, and the goal here is to test multiple number of clusters to find which one might be the best choice.

The tables that contain the references in the Annex for each one of the time series and each process are:

1. Time series with trend: Table 1
2. Stationary time series: Table 2
3. Time series with seasonality: Table 3

Each table will contain references to the following information:

- **First-phase granules clusterization**: Three-dimensional plot representing the center or cluster (second-phase granule) of each first-phase granule.
- **Fuzzy cognitive map values**: Matrix that represents the activation between the centers (second-phase granules).
- **Test granules membership clusterization:** Includes:
    - o Three-dimensional plot representing the center or cluster (second-phase granule) of each test first-phase granule.
    - o Test fuzzy triangle real values
- **Test granules membership function values:** Real values of the strength of the membership between the test first-phase granules and the second-phase granules.
- **Test granules membership function forecasted values:** Forecasted values of the strength of the membership between the test first-phase granules and the second-phase granules.
- **Test fuzzy triangle forecasted values:** Contains the forecasted values of the fuzzy triangle with the forecasting approach 1 and 2.
- **Error of type 2 and type 3**: Errors of the Equation 30 and Equation 31 respectively. Since there are two forecasting approaches, there are two errors of

type 3, for each approach. The goal here is to compare which approach has a lower error of type 3.

Additionally, the references of the summarize of the forecasted clusters and forecasted errors:

1. Time series with trend:
    a. Summary of forecasted clusters for scenario 1, 2 and 3: Table 52
    b. Summary of forecasted clusters for scenario 4, 5 and 6: Table 53
    c. Summary of errors of type 2 for scenario 1, 2 and 3: Table 54
    d. Summary of errors of type 2 for scenario 4, 5 and 6: Table 55
    e. Summary of errors of type 3 with method of forecasting 1 for scenario 1, 2 and 3: Table 56
    f. Summary of errors of type 3 with method of forecasting 1 for scenario 4, 5 and 6: Table 57
    g. Summary of errors of type 3 with method of forecasting 2 for scenario 1, 2 and 3: Table 58
    h. Summary of errors of type 3 with method of forecasting 2 for scenario 4, 5 and 6: Table 59
2. Stationary time series:
    a. Summary of forecasted clusters for scenario 1, 2 and 3: Table 108
    b. Summary of forecasted clusters for scenario 4, 5 and 6: Table 109
    c. Summary of errors of type 2 for scenario 1, 2 and 3: Table 110
    d. Summary of errors of type 2 for scenario 4, 5 and 6: Table 111
    e. Summary of errors of type 3 with method of forecasting 1 for scenario 1, 2 and 3: Table 112
    f. Summary of errors of type 3 with method of forecasting 1 for scenario 4, 5 and 6: Table 113
    g. Summary of errors of type 3 with method of forecasting 2 for scenario 1, 2 and 3: Table 114
    h. Summary of errors of type 3 with method of forecasting 2 for scenario 4, 5 and 6: Table 115
3. Time series with seasonality:
    a. Summary of forecasted clusters for scenario 1, 2 and 3: Table 164
    b. Summary of forecasted clusters for scenario 4, 5 and 6: Table 165
    c. Summary of errors of type 2 for scenario 1, 2 and 3: Table 166
    d. Summary of errors of type 2 for scenario 4, 5 and 6: Table 167
    e. Summary of errors of type 3 with method of forecasting 1 for scenario 1, 2 and 3: Table 168
    f. Summary of errors of type 3 with method of forecasting 1 for scenario 4, 5 and 6: Table 169
    g. Summary of errors of type 3 with method of forecasting 1 for scenario 1, 2 and 3: Table 170
    h. Summary of errors of type 3 with method of forecasting 1 for scenario 4, 5 and 6: Table 171

The rationale behind the assessment on which model has performed the best for each process is as follows:

1. Highest accuracy on forecasting the second-phase granule. If the model is not correctly forecasting the second-phase granule of the testing period, it translates into a wrong classification of the forecasted first-phase granule, and the forecast of the values of the first-phase granule might be biased since these forecasted values have a high dependency on the forecasted second-phase granule.
2. Highest accuracy on error type 2. The metric of the error of type 2 contains the deviation when forecasting the membership strength, which is basically computing the degree of belonging of each first-phase granule with all the second-phase granules. This point is quite related with the previous one, since the highest value of the forecasted membership strength will reflect the second-phase granule to which the first-phase granule belongs.
3. Highest accuracy on error of type 3: The metric of the error of type 3 contains the deviation when forecasting the values of $m, a,$ and b, which turns out to be the values of the first-phase granule. The evaluation here will be done with the approach of forecasting 1 (Equation 26, Equation 27 and Equation 28) and approach of forecasting 2 (Equation 36, Equation 37 and Equation 38).

4.2. Time series with trend

The selected time series with trend to be analyzed is the Gold Monthly Price - US Dollars per Troy Ounce from World Bank[1], from May 1999 till April 2024. This time series displays an upward trend in most of its time length, as shown in Figure 7.

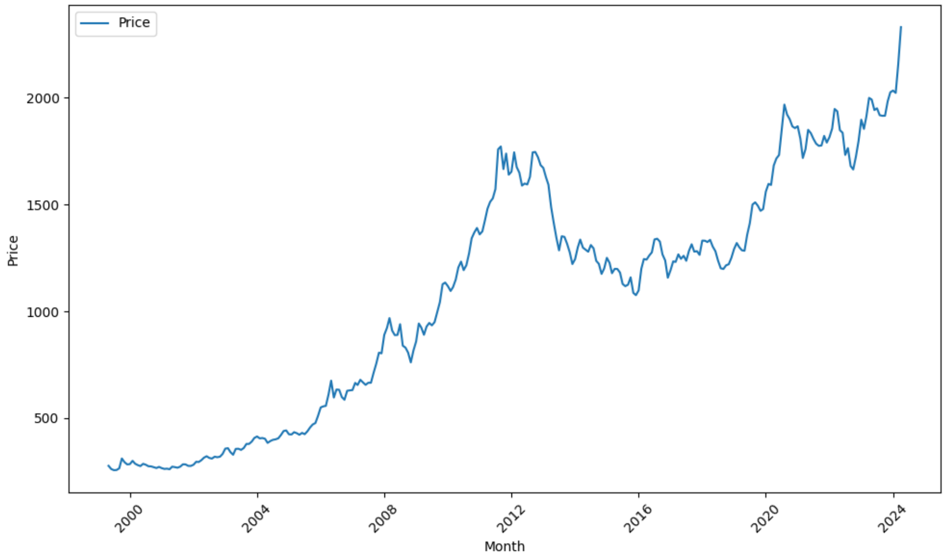


Figure 7 Trend time series

Since the data is monthly distributed, the interval selected for the granularization will be a year, and thus, each granule will summarize the information of 12 months. After applying the Principle of Justifiable Granularity, we yield to the first-phase granules shown in Figure 8.

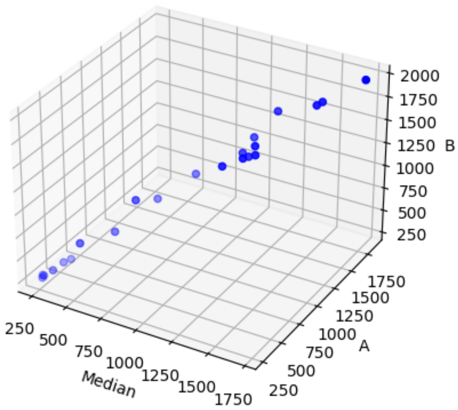


Figure 8 Trend time series transformed into first-phase granules

Since this time series has a trend, it can be easily appreciated in the first-phase granules clusterization graphs that there is a clear time clusterization of the granules as the time series advances, as we increase the number of clusters, and all the clusters are "ordered" in time.

[1] https://thedocs.worldbank.org/en/doc/5d903e848db1d1b83e0ec8f744e55570-0350012021/related/CMO-Historical-Data-Monthly.xlsx

As it can be observed in Figure 13 of the Annex, the Fuzzy C-Means error is minimized after less than a 100 iterations, which is indeed a high speed of convergence.

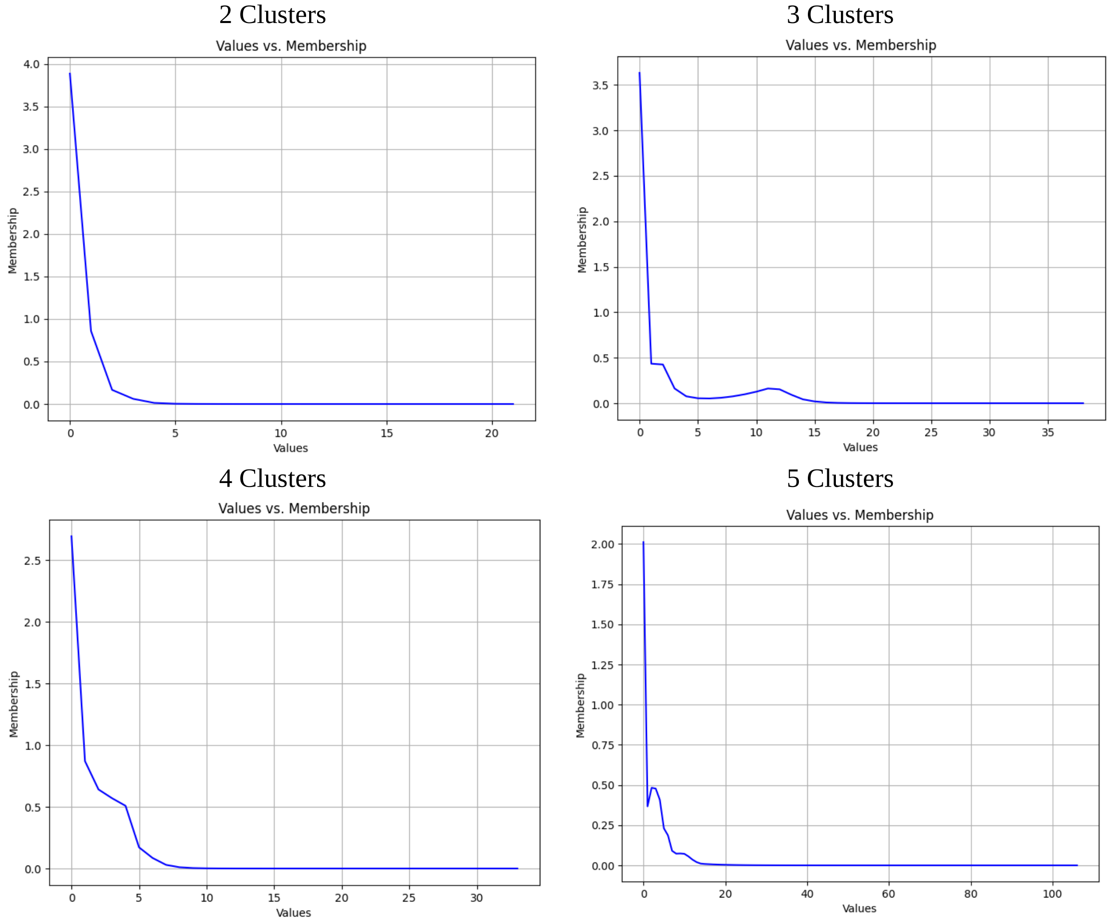
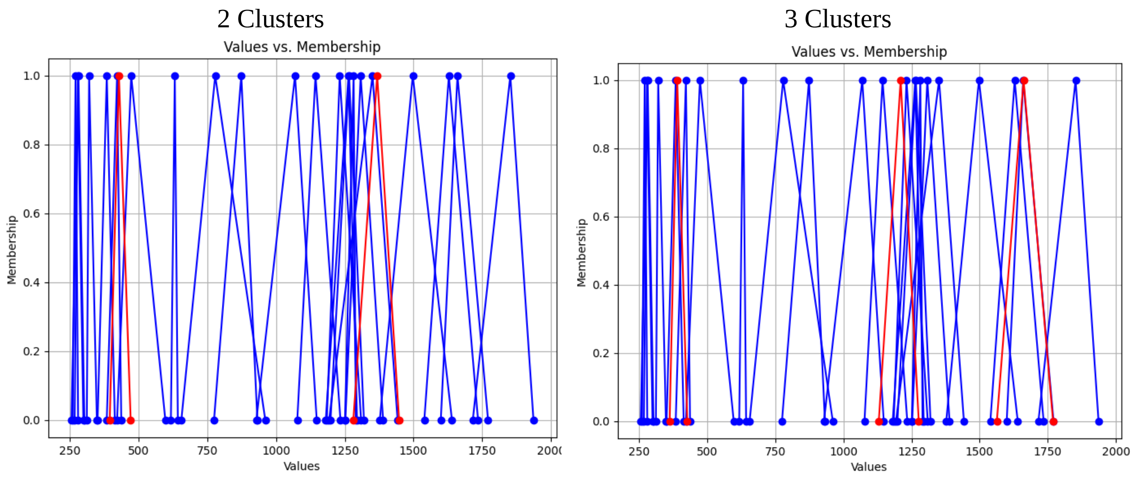


Figure 13 Trend time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

In Figure 14 of the Annex, the representation as fuzzy triangles of the first-phase granules and second-phase granules displays the emergence of second-phase granules in between the first-phase granules as the number of clusters increases. The structure of the graph and the nature of the trend time series suggests that more clusters could be considered.



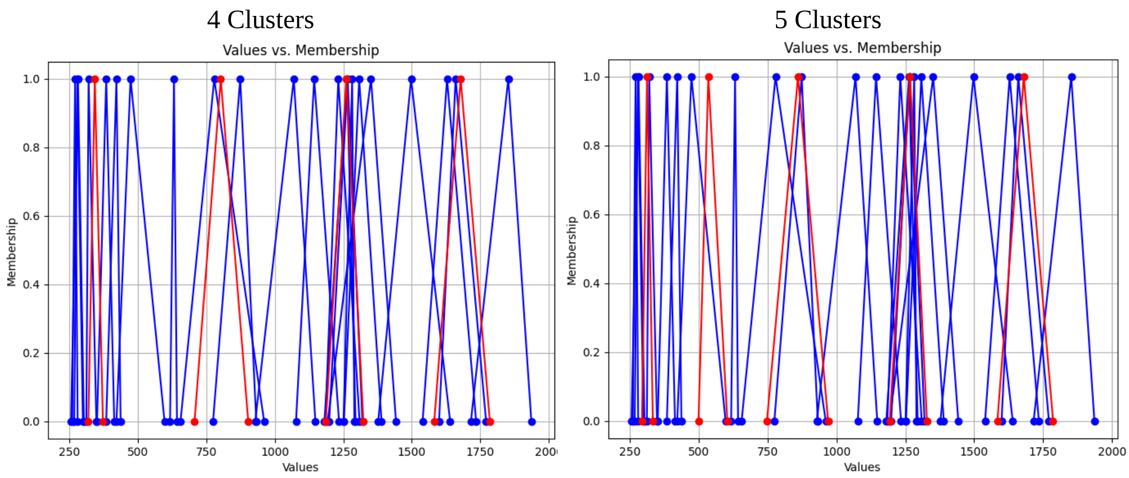


Figure 14 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the trend time series

An idea that lies behind of this process: if there were the same number of first and second-phase granules, the values of each optimized second-phase granule will match the values of the first-phase granules, yielding to an error of 0.

The following table contains the references of the different outcomes of the Scenarios:

Outcome	S. 1	S. 2	S. 3	S. 4	S. 5	S. 6
First-phase granules clusterization	Figure 15	Figure 17	Figure 19	Figure 21	Figure 23	Figure 25
Fuzzy cognitive map values	Table 4	Table 12	Table 20	Table 28	Table 36	Table 44
Test granules membership function values	Table 5	Table 13	Table 21	Table 29	Table 37	Table 45
Test fuzzy triangle forecasted values with method 1	Table 8	Table 16	Table 24	Table 32	Table 40	Table 48
Test fuzzy triangle forecasted values with method 2	Table 10	Table 18	Table 26	Table 34	Table 42	Table 50
Test granules membership clusterization	Table 6	Table 14	Table 22	Table 30	Table 38	Table 46
Test granules membership function forecasted values	Figure 16	Figure 18	Figure 20	Figure 22	Figure 24	Figure 26
Error of type 2	Table 7	Table 15	Table 23	Table 31	Table 39	Table 47
Error of type 3 (method of forecasting 1)	Table 9	Table 17	Table 25	Table 33	Table 41	Table 49
Error of type 3 (method of forecasting 2)	Table 11	Table 19	Table 27	Table 35	Table 43	Table 51

Table 1 Trend time series' objects references

The best model for each scenario are:

- Scenario 1: Model with 4 clusters.
- Scenario 2: Model with 4 clusters.
- **Scenario 3: Model with 4 clusters (best model).**
- Scenario 4: Model with 3 clusters.
- Scenario 5: Model with 5 clusters.
- Scenario 6: Model with 3 clusters.

The Scenario that is going to be commented below is the Scenario 3, which corresponds with the Scenario with the highest accuracy of the error of type 3, and ultimately, the Scenario that has predicted the values of the next three first-phase granules (next three periods) with lowest error. This model is optimized by a genetic algorithm (proposed by the original article), and the strength of the membership is measured by the function proposed in this article (Equation 35). In this case, the first-phase granule values forecasted by the proposed approach has yielded to a lower error than the values forecasted by the approach proposed by the original article.

The clusterization of the first-phase granules of the best model (S3) can be seen in the Figure 19 of the Annex.

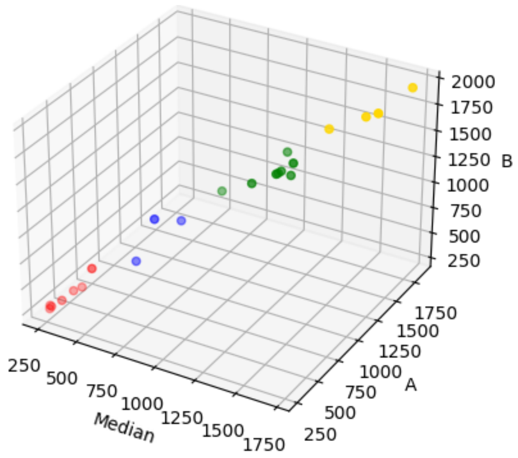


Figure 19 First-phase granules of trend time series by applying Scenario 3

Overall, the genetic algorithm optimization has performed better than the optimization with IPOPT. This is observed in the increase of the error of type 2 for the IPOPT optimization algorithm (see in the Annex Table 54 for genetic algorithm optimization and Table 55 for IPOPT optimization), specially, when the number of clusters increases, which in the case of the trend time series is the opposite with the genetic algorithm optimization, with exception of the S2. This means that the IPOPT optimization algorithm deals worse as the number of clusters increase. The conclussion of this point would be that for trend time series the optimization with genetic optimization algorithm should be chosen over IPOPT optimization algorithm.

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,065997	0,470557	0,356979
Test granule 2	0,080320	0,545939	0,438391
Test granule 3	0,080393	0,668784	0,410233
3 CLUSTERS			
Test granule 1	0,037497	0,634404	0,107196
Test granule 2	0,031341	0,657661	0,092908
Test granule 3	0,036219	0,772574	0,074141
4 CLUSTERS			
Test granule 1	0,014354	0,856382	**0,011165**
Test granule 2	0,012928	1,025407	**0,006167**
Test granule 3	0,016770	1,193808	**0,007953**

5 CLUSTERS			
Test granule 1	0,025723	1,101291	0,004173
Test granule 2	0,017681	1,469028	0,001369
Test granule 3	0,021003	1,696927	0,004283

Table 54 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,124977	0,541255	0,621521
Test granule 2	0,125041	0,576098	0,629831
Test granule 3	0,125041	0,609447	0,584727
3 CLUSTERS			
Test granule 1	0,159826	0,354419	0,124860
Test granule 2	0,159827	0,421372	0,126770
Test granule 3	0,159827	0,422313	0,109122
4 CLUSTERS			
Test granule 1	0,306587	0,309686	0,260548
Test granule 2	0,306600	0,309714	0,260982
Test granule 3	0,306600	0,309714	0,256794
5 CLUSTERS			
Test granule 1	0,502717	0,505731	0,453781
Test granule 2	0,502771	0,505793	0,453968
Test granule 3	0,502771	0,505793	0,452591

Table 55 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the trend time series

The Scenario that had the highest accuracy when forecasting the membership strength has been S3 (**proposed function in this article**), with an average error of type 2 lower than 0,01 for the three forecasted first-phase granules (see in Annex Table 54). S1 has the second best accuracy with an average value slightly over 0,01. Also S3 with 5 clusters should be commented at this point, since it has the lowest value, but it has forecasted correctly only one second-phase granule.

When reviewing the forecasted values of the first-phase granule, $X(x; m, a, b)$, there is a significant difference in the error of type 3 between the approach of forecasting proposed by the original article and the proposed approach of forecasting (see in Annex Table 56 vs Table 58, and Table 57 vs Table 59), where **the proposed approach of forecasting is yielding to much better results**:

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,462086759	0,54851	0,481712072
Test granule 2	0,433597199	0,503135822	0,484736405
Test granule 3	0,494646807	0,559422848	0,540624856

3 CLUSTERS			
Test granule 1	0,355447569	0,461642523	0,410274911
Test granule 2	0,349344874	0,48446466	0,413616032
Test granule 3	0,423622644	0,533210102	0,476801738
4 CLUSTERS			
Test granule 1	0,374223806	0,404145991	0,412463195
Test granule 2	0,36755856	0,41258381	0,41485382
Test granule 3	0,439399957	0,473504033	0,481636351
5 CLUSTERS			
Test granule 1	0,432022914	0,499670408	0,46953166
Test granule 2	0,413264743	0,511131022	0,466206374
Test granule 3	0,412422673	0,553767656	0,527003919

Table 56 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,335195563	0,786401453	0,247820827
Test granule 2	0,328935266	0,280964904	0,240737744
Test granule 3	0,401226953	0,807617029	0,322530672
3 CLUSTERS			
Test granule 1	0,20556777	0,788752014	0,788751385
Test granule 2	0,1980868	0,786762745	0,786762109
Test granule 3	0,28447438	0,809734122	0,809733555
4 CLUSTERS			
Test granule 1	0,180036751	0,088920651	**0,088919742**
Test granule 2	0,172315362	0,080341245	**0,080340327**
Test granule 3	0,261479217	0,179413182	**0,179412363**
5 CLUSTERS			
Test granule 1	0,179926357	0,830948044	0,088719317
Test granule 2	0,341482679	0,829356124	0,304392767
Test granule 3	0,412422673	0,84773905	0,37932834

Table 58 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,423639657	0,548406434	0,509819034
Test granule 2	0,418188306	0,503124816	0,504095918
Test granule 3	0,480865044	0,559388929	0,557518101
3 CLUSTERS			
Test granule 1	0,405391814	0,449288671	0,405391814
Test granule 2	0,399792537	0,452763595	0,399792537
Test granule 3	0,464450994	0,511684467	0,464450994

4 CLUSTERS			
Test granule 1	0,441362749	0,441364571	0,441362749
Test granule 2	0,436102201	0,43610404	0,436102201
Test granule 3	0,496849133	0,496850774	0,496849133
5 CLUSTERS			
Test granule 1	0,48995321	0,48995321	0,489951388
Test granule 2	0,485150227	0,485146548	0,485148387
Test granule 3	0,540613369	0,540610087	0,540611728

Table 57 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,335188181	0,786400939	0,742810186
Test granule 2	0,328927815	0,280961244	0,240737744
Test granule 3	0,401220305	0,807616566	0,322530672
3 CLUSTERS			
Test granule 1	0,20556746	0,788751385	0,090796455
Test granule 2	0,198086487	0,786762109	0,082234712
Test granule 3	0,284474101	0,809733555	0,181102672
4 CLUSTERS			
Test granule 1	0,835561702	0,808737464	0,577225818
Test granule 2	0,698732049	0,806936393	0,080340327
Test granule 3	0,731186695	0,827734526	0,179412363
5 CLUSTERS			
Test granule 1	0,843425675	0,541203866	0,694924859
Test granule 2	0,341489612	0,536883497	0,692052042
Test granule 3	0,412428859	0,586773577	0,725226304

Table 59 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the trend time series

The real values of the forecasted first-phase granules can be found in the Annex, Figure 20.

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 20 Test first-phase granules values and 3D representation of Scenario 3 applied to trend time series

the forecasted values by the approach proposed by the original article for each combination of the number of clusters can be found in the Annex, Table 24.

2 clusters

	a	m	b
Test granule 1	885,15	948,08	1011,3
Test granule 2	871,56	933,68	996,31
Test granule 3	870,83	932,9	995,5

3 clusters

	a	m	b
Test granule 1	1010	1078,3	1148,3
Test granule 2	994,74	1062,1	1131,4
Test granule 3	994,74	1062	1131,4

4 clusters

	a	m	b
Test granule 1	998,19	1074,2	1152,2
Test granule 2	984,21	1059,8	1137,5
Test granule 3	977,18	1052,1	1129,4

5 clusters

	a	m	b
Test granule 1	899,5	968,48	1043,4
Test granule 2	896,67	965,53	1040,1
Test granule 3	890,38	958,85	1033

Table 24 Test fuzzy triangles with forecasting method 1 for Scenario 3 applied to trend time series

The forecasted values by the proposed approach for each combination of the number of clusters can be found in the Annex, Table 26.

2 clusters

	a	m	b
Test granule 1	1290,8	1378,8	1458,6
Test granule 2	1290,8	1378,8	1458,6
Test granule 3	1290,8	1378,8	1458,6

3 clusters

	a	m	b
Test granule 1	359,22	383,62	416,56
Test granule 2	359,22	383,62	416,56
Test granule 3	359,22	383,62	416,56

4 clusters

	a	m	b
Test granule 1	**1562,9**	**1663,6**	**1773,8**
Test granule 2	**1562,9**	**1663,6**	**1773,8**
Test granule 3	**1562,9**	**1663,6**	**1773,8**

5 clusters

	a	m	b
Test granule 1	1563,4	1663,9	1774,1
Test granule 2	1191,5	1264,3	1326,3
Test granule 3	1191,5	1264,3	1326,3

Table 26 Test fuzzy triangles with forecasting method 2 for Scenario 3 applied to trend time series

It can be observed that the forecast with the lowest error is achieved by the model with 4 clusters forecasted with the **proposed approach**.

As stated before, this is because the approach to forecast the values first-phase granules proposed by the original article is a weighted average of the values by the forecasted membership strength, which means that forecasting the right second-phase granule is not as important since each forecasted value of the first-phase granules will contain a portion of information of each second-phase granule.

When considering the proposed forecasting approach the situation is the opposite. Forecasting the correct second-phase granule becomes a heavily important part of the process since the forecasted values of each first-phase granule will be averages of the values of all the first-phase granules that belong to that second-phase granule.

The highest accuracy of the approach to forecast the values first-phase granules proposed by the original article is reached by the S1 with 3 clusters, with an average error of type 3 of 0,37 (see Table 58), whereas the highest accuracy of the proposed approach is reached with S2 and S3 with an average error of type 3 of 0,11 (see Table 58). This result is expected since the weighted forecasted values of theapproach proposed by the original article adds information of the lowest values of the time series. If there is a clear trend there is a high chance that the forecasted values will be either lower or higher than the last values of the train partition, depending on the trend being downwards or upwards respectively. If the second-phase granule is correctly forecasted, the forecasted values of the first-phase granule with the proposed approach will be computed as an average of the values of the first-phase granule that are clustered with the corresponding second-phase granule, and thus, the values will be much closer to the real values since the first-phase granules that belong to other stages of the trend time series will not contribute on the forecasted values.

The biggest issue that arises when dealing with a trend time series is the fact that the increase of the values due to the upward trend is not included in the forecast, by any forecasting aproach. This might suggest that trend time series forecasted values will mostly be slightly spurious, and methodologies to deal with the trend should be considered as a previous steps to the modeling. In the next section the first differences of the trend time series is considered, but detrending the series and adding an increase to the forecasted values is another approach that could be considered.

4.3. Stationary time series

The stationary time series that will be analyzed will be first differences of the previous example (Gold Monthly Price - US Dollars per Troy Ounce from World Bank, from May 1999 till April 2024). After applying the Dickey-Fuller test[2], the series is stationary in first differences. The shape of the serie is shown in Figure 9.

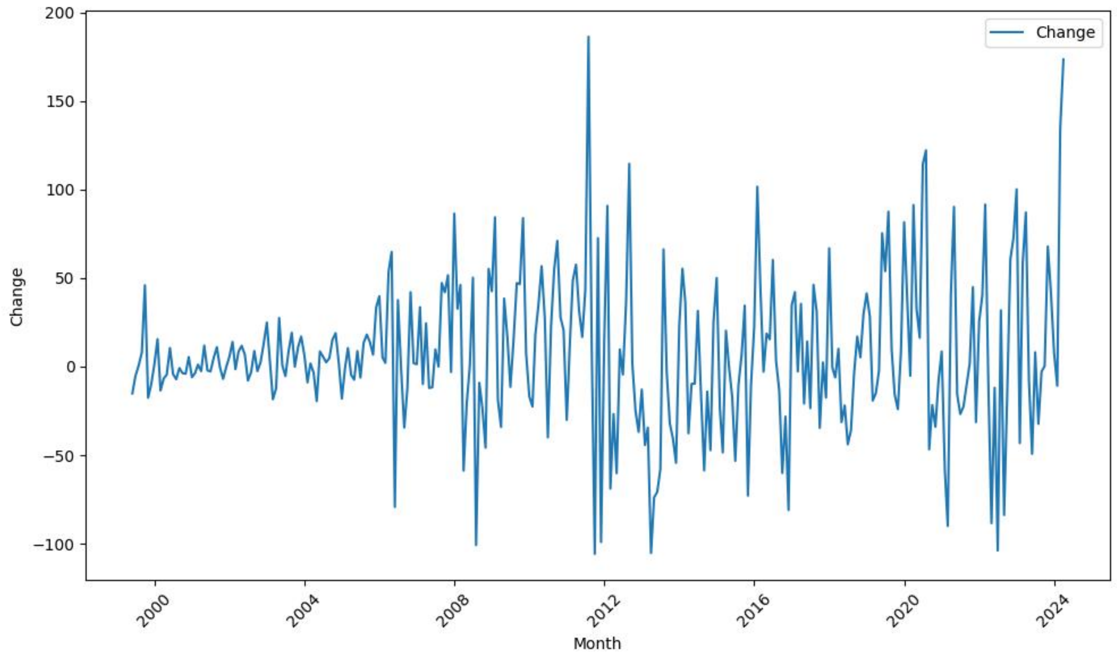


Figure 9 Stationary time series

In the previous example, using the same time series but without taking the first differences, intervals of 12 months were chosen. For this example we will keep the same granularity to assess if the model works better in stationary series, when comparing with trend time series. After applying the Principle of Justifiable Granularity, we yield to the first-phase granules shown in Figure 10.

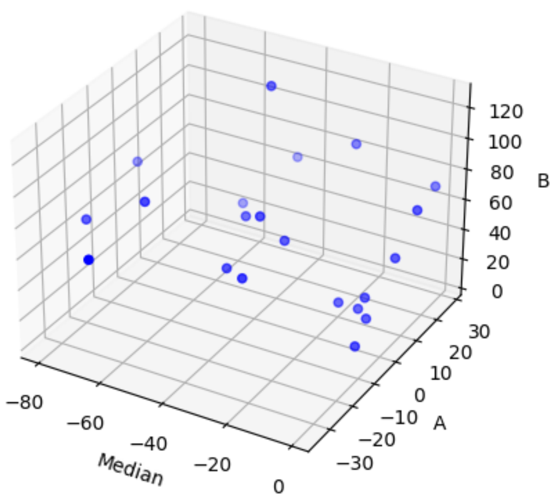


Figure 10 Stationary time series transformed into first-phase granules

[2] The Dickey-Fuller test has been computed with intercept and no trend

As it could be expected, the position of the first-phase granules in the 3D chart looks much more disorganized, when compared to Figure 8. This shows that the essence of a stationary time series translates to the location of the points in the space, yielding to a random structure.

As it can be observed in Figure 27, the Fuzzy C-Means error is minimized with more iterations than the previous case study, but still having a high speed of convergence minimizing the error with less than 200 iterations:

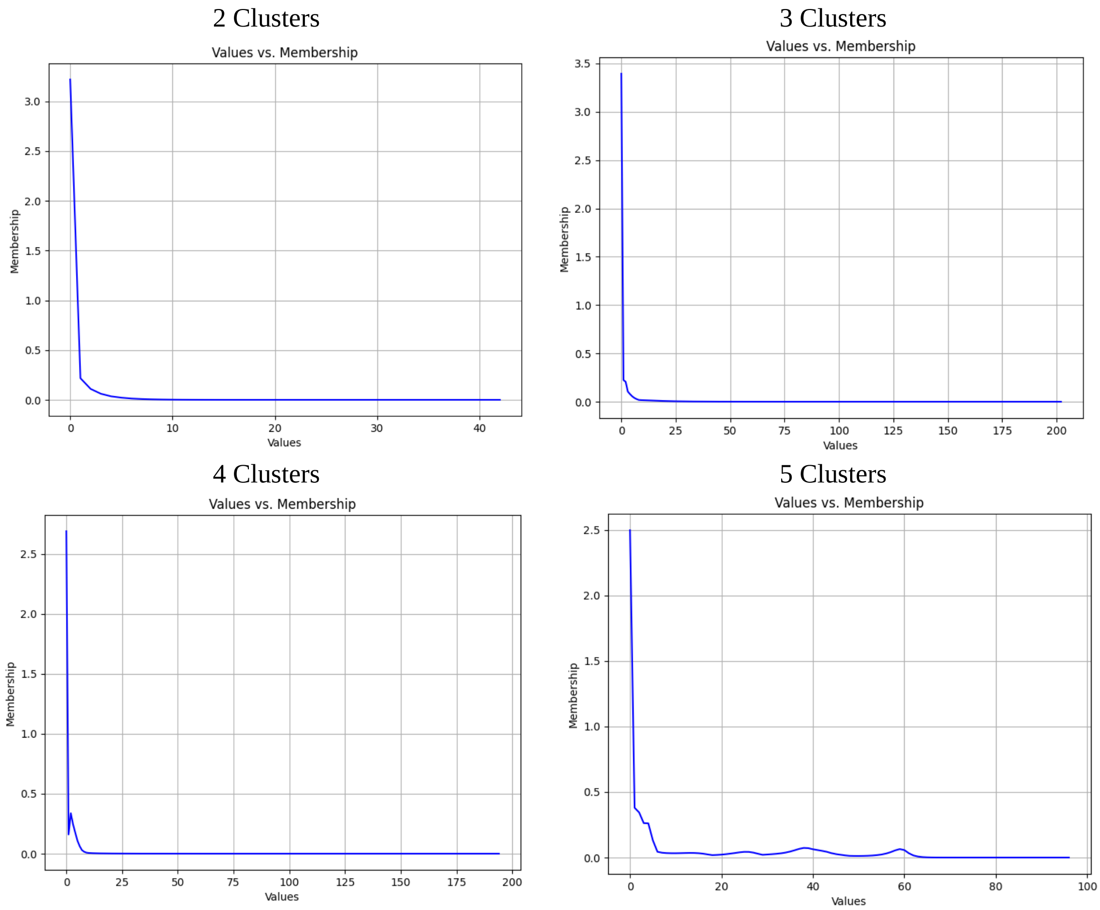


Figure 27 Stationary time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

In Figure 28, the representation as fuzzy triangles of the first-phase granules and second-phase granules displays an opposite conclussion with respect to the trend case study since visually using more than 3 clusters seems to saturate the distribution of the centers:

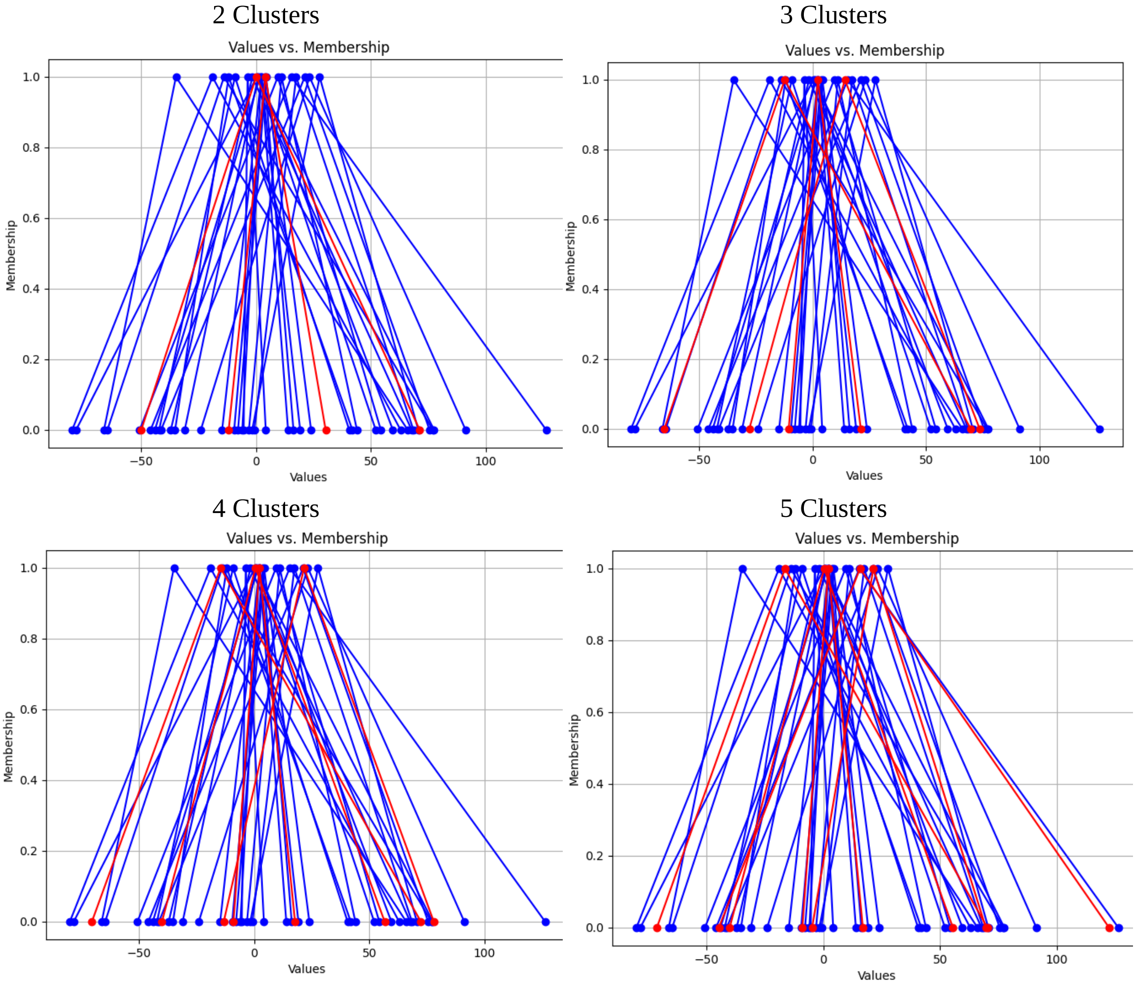


Figure 28 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the stationary time series

The following table contains the references of the different outcomes of the Scenarios:

Outcome	S. 1	S. 2	S. 3	S. 4	S. 5	S. 6
First-phase granules clusterization	Figure 29	Figure 31	Figure 33	Figure 35	Figure 37	Figure 39
Fuzzy cognitive map values	Table 60	Table 68	Table 76	Table 84	Table 92	Table 100
Test granules membership function values	Table 61	Table 69	Table 77	Table 85	Table 93	Table 101
Test fuzzy triangle forecasted values with method 1	Table 64	Table 72	Table 80	Table 88	Table 96	Table 104
Test fuzzy triangle forecasted values with method 2	Table 66	Table 74	Table 82	Table 90	Table 98	Table 106
Test granules membership clusterization	Table 62	Table 70	Table 78	Table 86	Table 94	Table 102
Test granules membership function forecasted values	Figure 30	Figure 32	Figure 34	Figure 36	Figure 38	Figure 40
Error of type 2	Table 63	Table 71	Table 79	Table 87	Table 95	Table 103
Error of type 3 (method of forecasting 1)	Table 65	Table 73	Table 81	Table 89	Table 97	Table 105
Error of type 3 (method of forecasting 2)	Table 67	Table 75	Table 83	Table 91	Table 99	Table 107

Table 2 Stationary time series' objects references

The best model for each Scenario are:

- Scenario 1: Model with 2 clusters.
- Scenario 2: Model with 5 clusters.
- Scenario 3: Model with 5 clusters.
- Scenario 4: Model with 5 clusters.
- **Scenario 5: Model with 5 clusters (best model).**
- Scenario 6: Model with 5 clusters.

The Scenario that is going to be commented below is the Scenario 5, which corresponds with the Scenario with the highest accuracy of the error of type 3, and ultimately, the Scenario that has predicted the values of the next three first-phase granules (next three periods) with lowest error. This model is optimized by the IPOPT optimization algorithm (**proposed optimization algorithm**), and the strength of the membership is measured by the Match function proposed in the original article (Equation 17). In this case, the first-phase granule values forecasted by the approach proposed by the original article has yielded to a lower error than the values forecasted by the proposed approach.

~~The clusterization of the first-phase granules of the best model (S5):~~

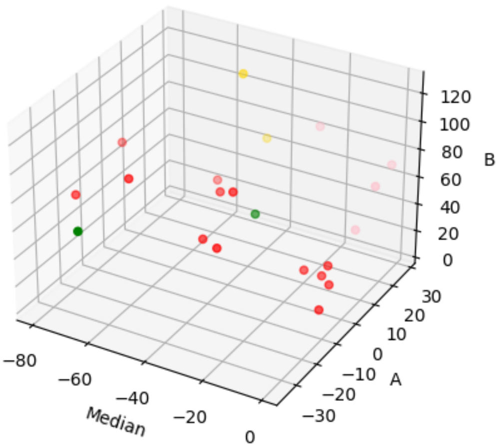


Figure 37 First-phase granules of stationary time series by applying Scenario 5

Overall the results are worse than the ones shown for the trend series. In this serie, the genetic algorithm optimization is still performing better than the IPOPT optimization when considering the error of type 2 (Table 110 vs Table 111), but the difference is smaller than in the last case study:

	Scenario 1	Scenario 2	Scenario 3
		2 CLUSTERS	
Test granule 1	0,027419	1,132805	0,485258
Test granule 2	0,036543	0,004821	0,434316
Test granule 3	0,027165	0,183543	0,456329
		3 CLUSTERS	
Test granule 1	0,362202	9,254288	0,055580
Test granule 2	0,415635	0,390972	0,039547
Test granule 3	0,333091	2,537957	0,047841

4 CLUSTERS			
Test granule 1	0,467926	18,893930	0,010791
Test granule 2	0,574787	0,989967	0,006295
Test granule 3	0,567610	2,092913	0,011785
5 CLUSTERS			
Test granule 1	0,692287	20,859655	0,005585
Test granule 2	1,088974	1,725956	0,002410
Test granule 3	1,019136	1,968028	0,006410

Table 110 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,071007	1,133030	0,492386
Test granule 2	0,070289	0,004628	0,523320
Test granule 3	0,075108	0,184067	0,532664
3 CLUSTERS			
Test granule 1	0,758534	11,954376	0,098221
Test granule 2	0,763201	0,775294	0,092386
Test granule 3	0,763201	3,373756	0,096541
4 CLUSTERS			
Test granule 1	1,433545	18,963287	0,254158
Test granule 2	1,462879	2,042927	0,252806
Test granule 3	1,462880	3,034555	0,253587
5 CLUSTERS			
Test granule 1	2,203769	**20,049126**	0,451329
Test granule 2	2,282947	**2,917830**	0,451374
Test granule 3	2,283169	**3,201332**	0,451659

Table 111 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the stationary time series

The highest accuracy when forecasting the membership strength has been S3 (**proposed function to measure the membership strength**) with 5 clusters (average error of type 2 below 0,01) and 4 clusters (average error slightly below 0,01), which can be observed in Table 110.

This is an important outcome since the proposed function to forecast the membership strength has performed the best for the trend time series and its first differences, altough it is not correctly forecasting the second-phase granule. Some modifications of the FCM optimization could be considered here, which may imply modifications in Equation 19 and Equation 20. Overall, the forecast of the second-phase granules has been performed worse than the last case study. This could be due to a wrong specification of the length of the periods considered for each first-phase granule, so testing the models with periods of shorter length should be considered here.

The impact of wrongly forecasting the second-phase granules has a deep impact in the error of type 3 measured by the proposed forecasting approach, since the forecast of the values of the first-phase granules is done with the average of the values of the first-phase granules that belong to an incorrect second-phase granule. This is the reason of why the errors of type 3 of the forecasting approach proposed by the original article are lower than the errors of type 3 of the proposed forecasting approach (Table 112 vs Table 114 and Table 113 vs Table 115). The error of type 3 of the best model is 0,31, which is a quite high value.

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,253650357	0,300284039	0,351906725
Test granule 2	0,508571495	0,618280746	0,622990698
Test granule 3	0,424327165	0,421879522	0,421740157
3 CLUSTERS			
Test granule 1	0,291861829	0,266089426	0,273281107
Test granule 2	0,668916935	0,532192306	0,594212125
Test granule 3	0,440850659	0,484511461	0,43148732
4 CLUSTERS			
Test granule 1	0,367090892	0,24071926	0,257549936
Test granule 2	0,651213948	0,465635956	0,581581826
Test granule 3	0,437432793	0,444697281	0,407641459
5 CLUSTERS			
Test granule 1	0,202193513	0,162264065	0,176498992
Test granule 2	0,634511756	0,426584864	0,509219838
Test granule 3	0,467629274	0,357463255	0,313780367

Table 112 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,254660986	0,28509529	0,75658512
Test granule 2	0,572786893	0,514517365	0,843473566
Test granule 3	0,404967634	0,410438657	0,744987434
3 CLUSTERS			
Test granule 1	0,730568078	0,375692688	0,784044347
Test granule 2	0,840732015	0,654789417	0,872190904
Test granule 3	0,740520905	0,602643363	0,791773665
4 CLUSTERS			
Test granule 1	0,7763698	0,357522448	0,257000183
Test granule 2	0,864349086	0,661805263	0,876129575
Test granule 3	0,778997791	0,584570863	0,798190541

5 CLUSTERS			
Test granule 1	0,2992954	0,349276159	0,599239509
Test granule 2	0,551679989	0,624643575	0,876895714
Test granule 3	0,469262813	0,572309801	0,799438733

Table 114 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,271804105	0,300322522	0,31394081
Test granule 2	0,58083719	0,618435937	0,610203805
Test granule 3	0,422752266	0,421871906	0,422078288
3 CLUSTERS			
Test granule 1	0,327170607	0,285972146	0,26440077
Test granule 2	0,641507502	0,598623381	0,575399897
Test granule 3	0,440421141	0,482229076	0,423998934
4 CLUSTERS			
Test granule 1	0,285674363	0,240261132	0,248928899
Test granule 2	0,615550414	0,55914458	0,570849343
Test granule 3	0,42393344	0,391215444	0,395033128
5 CLUSTERS			
Test granule 1	0,25161444	**0,157588419**	0,177741433
Test granule 2	0,563427757	**0,480946571**	0,506904595
Test granule 3	0,331413449	**0,300913868**	0,311689133

Table 113 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,254660986	0,28509529	0,75658512
Test granule 2	0,572786893	0,514517365	0,843473566
Test granule 3	0,404967634	0,410438657	0,744987434
3 CLUSTERS			
Test granule 1	0,730568078	0,375692688	0,784044347
Test granule 2	0,840732015	0,654789417	0,872190904
Test granule 3	0,740520905	0,602643363	0,791773665
4 CLUSTERS			
Test granule 1	0,7763698	0,357522448	0,306380795
Test granule 2	0,54556397	0,661805263	0,626010377
Test granule 3	0,778997791	0,584570863	0,545033889

5 CLUSTERS

Test granule 1	0,782371266	0,492303463	0,599239509
Test granule 2	0,659659702	0,624643575	0,256022998
Test granule 3	0,46928566	0,572309801	0,264953164

Table 115 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the stationary time series

The real values of the forecasted first-phase granules can be found in the Annex, Figure 38.

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 38 Test first-phase granules values and 3D representation of pr ocess 5 applied to stationary time series

The forecasted values for each combination of the number of clusters by the approach proposed in the original article can be found in the Annex, Table 96.

2 clusters

	a	m	b
Test granule 1	-29,429	2,2502	49,184
Test granule 2	-29,817	2,209	49,602
Test granule 3	-29,817	2,209	49,602

3 clusters

	a	m	b
Test granule 1	-36,27	-1,116	51,643
Test granule 2	-35,899	-1,1195	51,087
Test granule 3	-35,899	-1,1195	51,087

4 clusters

	a	m	b
Test granule 1	-36,155	1,7611	59,394
Test granule 2	-34,27	2,3842	57,658
Test granule 3	-34,178	2,4665	57,731

5 clusters

	a	m	b
Test granule 1	**-33,878**	**5,7672**	**70,146**
Test granule 2	**-35,788**	**4,7971**	**70,456**
Test granule 3	**-35,565**	**4,795**	**70,077**

Table 96 Test fuzzy triangles with forecasting method 1 for Scenario 5 applied to stationary time series

The forecasted values by the proposed approach for each combination of the number of clusters can be found in the Annex, Table 98.

2 clusters

	a	m	b
Test granule 1	-40,182	3,4553	60,222
Test granule 2	-40,182	3,4553	60,222
Test granule 3	-40,182	3,4553	60,222

3 clusters

	a	m	b
Test granule 1	-37,797	-6,2161	42,27
Test granule 2	-37,797	-6,2161	42,27
Test granule 3	-37,797	-6,2161	42,27

4 clusters

	a	m	b
Test granule 1	-35,86	-4,025	40,515
Test granule 2	-35,86	-4,025	40,515
Test granule 3	-35,86	-4,025	40,515

5 clusters

	a	m	b
Test granule 1	-6,86	20,2	68,75
Test granule 2	-39,03	-3,67	44,94
Test granule 3	-39,03	-3,67	44,94

Table 98 Test fuzzy triangles with forecasting method 2 for Scenario 5 applied to stationary time series

It can be observed that the forecast with the lowest error is achieved by the model with 4 clusters forecasted with the forecasting approach proposed by the original article.

As a comment, S1 and S3 with 2 clusters have correctly forecasted the second-phase granule, but they still have high errors of type 2 and 3. This is because eventhough they forecasted they cluster well, the forecast of the membership strength was not good, and probably two clusters are not enough to correctly cluster this time series and thus, the forecasts are made based on groups that are not well defined.

This implies that FCMs to model granular series might not be a good choice for stationary time series, although, more studies should be done by considering a different period length, or a modification of Formulas 20 and 21.

4.4. Time series with seasonality

The selected time series with seasonality to be analyzed is maximum temperature per month recored at el Puerto de Navacerrada from 2000 till 2012[3]. This time series displays a strong seasonality and according with the Dickey-Fuller test the serie is stationary with a 10% of significance. The time serie can be observed in in Figure 11:

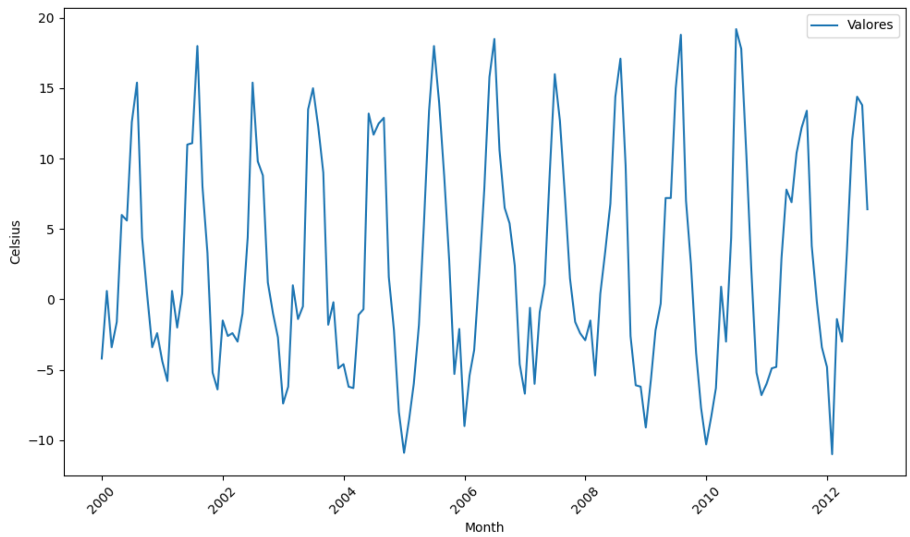


Figure 11 Seasonality time series

In this example we also have monthly data, but we are analyzing the maximum temperature and the best interval size here could be either 3 (to match the seasons) but finally 4 are chosen, due to the fact that 3 data points might be not enough to correctly optimize the granularization process. Since this time series can also be considered stationary at a 10%, it can also be considered a variation of the stationary time series case study with a shorter period length.

After applying the Principle of Justifiable Granularity, we yield to the first-phase granules shown Figure 12. It can easily be checked in the graph that there is a very visual split in between cold periods and warm periods, with no data between the two groups.

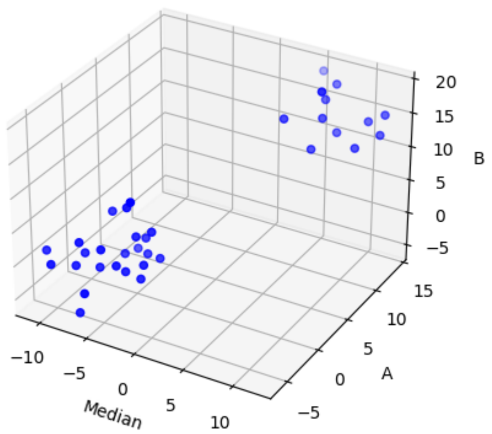


Figure 12 Seasonality time series transformed into first-phase granules

[3] https://www.madrid.org/iestadis//fijas/coyuntu/otros/cltempe.htm

As it can be observed in the Annex, Figure 41, the speed of convergence is slightly faster than the stationary time series case study but slower than the trend time series case study.:

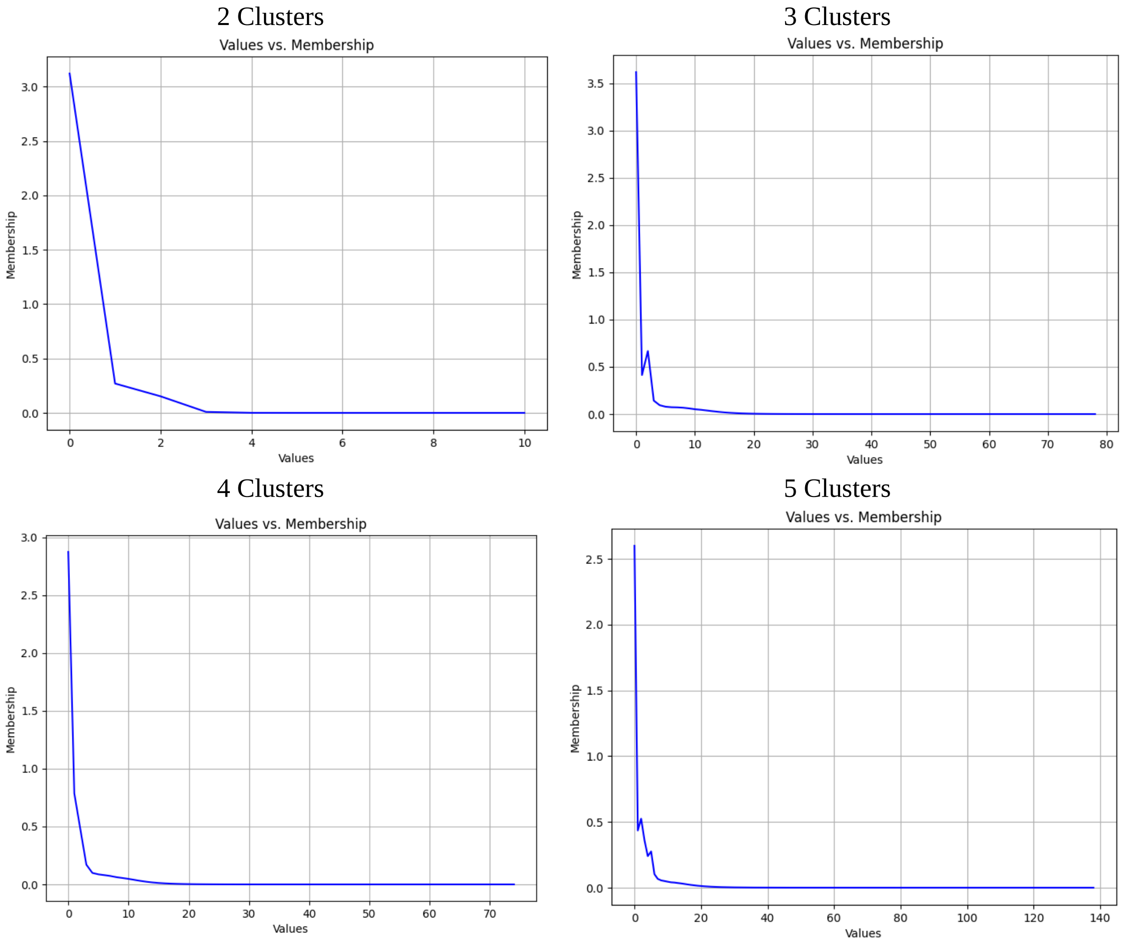


Figure 41 Seasonality time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

In Figure 42, the split between warm and cold can be easily appreciated. The second-phase granules are mostly found in the cold group when increasing the number of clusters in the optimization. It would be interesting to analye the behaviour of a warm location to check if the second-place granules would be mostly found in the warm group.

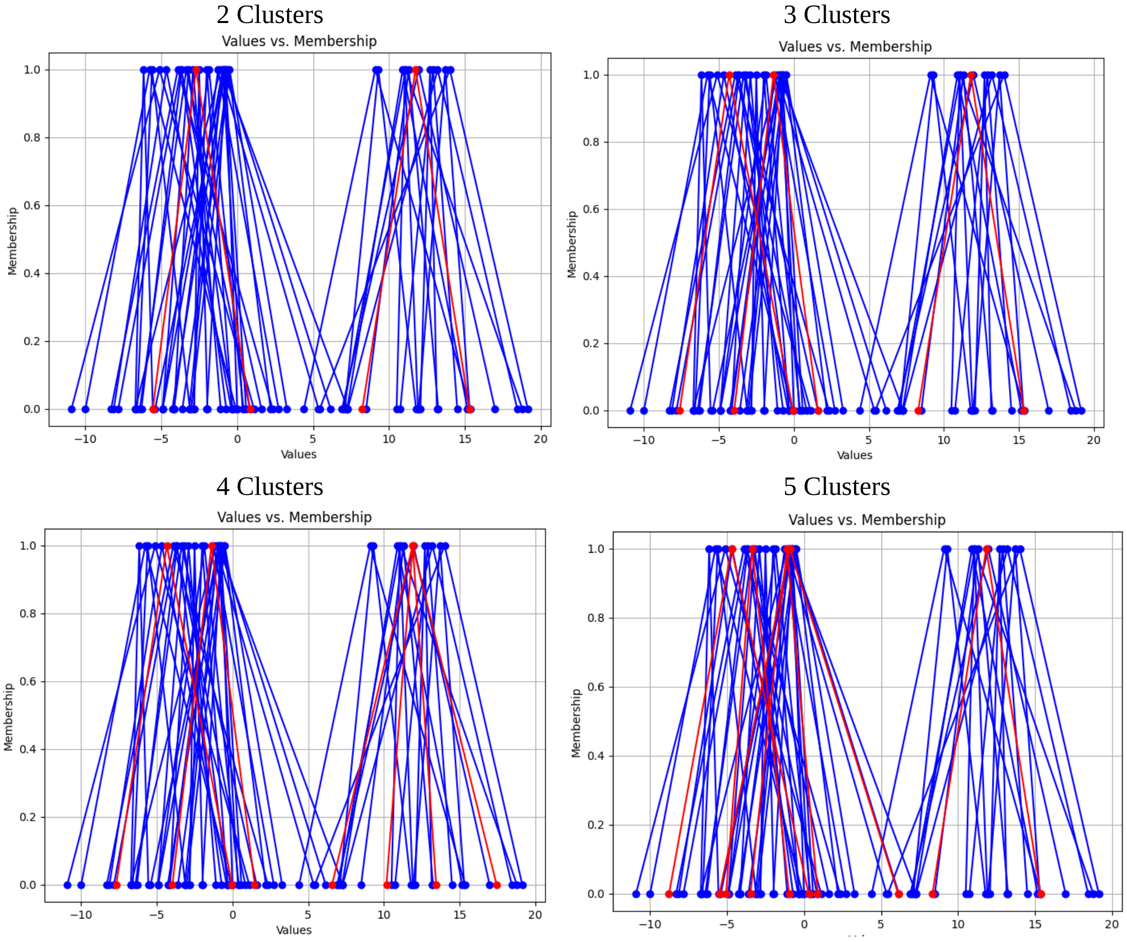


Figure 42 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the seasonality time series

The following table contains the references of the different outcomes of the Scenarios:

Outcome	S. 1	S. 2	S. 3	S. 4	S. 5	S. 6
First-phase granules clusterization	Figure 43	Figure 45	Figure 47	Figure 49	Figure 51	Figure 53
Fuzzy cognitive map values	Table 116	Table 124	Table 132	Table 140	Table 148	Table 156
Test granules membership function values	Table 117	Table 125	Table 133	Table 141	Table 149	Table 157
Test fuzzy triangle forecasted values with method 1	Table 120	Table 128	Table 136	Table 144	Table 152	Table 160
Test fuzzy triangle forecasted values with method 2	Table 122	Table 130	Table 138	Table 146	Table 154	Table 162
Test granules membership clusterization	Table 118	Table 126	Table 134	Table 142	Table 150	Table 158
Test granules membership function forecasted values	Figure 44	Figure 46	Figure 48	Figure 50	Figure 52	Figure 54
Error of type 2	Table 119	Table 127	Table 135	Table 143	Table 151	Table 159
Error of type 3 (method of forecasting 1)	Table 121	Table 129	Table 137	Table 145	Table 153	Table 161
Error of type 3 (method of forecasting 2)	Table 123	Table 131	Table 139	Table 147	Table 155	Table 163

Table 3 Seasonality time series' objects references

The best model for each Scenario:

- Scenario 1: Model with 3 clusters.
- Scenario 2: Model with 4 clusters.
- **Scenario 3: Model with 3 clusters (best model).**
- Scenario 4: Model with 3 clusters.
- Scenario 5: Model with 2 clusters.
- Scenario 6: Model with 5 clusters.

The Scenario that is going to be commented below is the Scenario 3, which corresponds with the Scenario with the highest accuracy of the error of type 3, and ultimately, the Scenario that has predicted the values of the next three first-phase granules (next three periods) with lowest error. This model is optimized by a genetic algorithm (proposed by the original article) and the strength of the membership is measured by the **function proposed in this article** (Equation 35). In this case, the first-phase granule values forecasted by the proposed approach has yielded to a lower error than the values forecasted by the approach proposed by the original article.

The clusterization of the first-phase granules of the best model (S3) can be seen in.

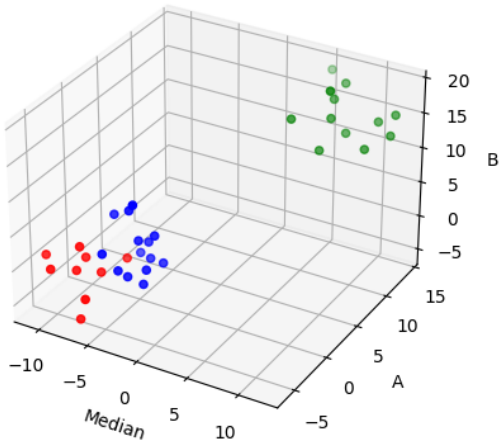


Figure 47 First-phase granules of seasonality time series by applying Scenario 3

In this case study, the genetic optimization algorithm is still working slightly better overall than the IPOPT optimization (see in Annex, Table 166 vs Table 167). The lowest error of type 2 is reached again by the proposed membership function to with genetic optimization (Scenario 3) (See in Annex ,Table 166). Eventhough the error of type 2 is quite low. As an important outcome, no Scenario is correctly forecasting the second-phase granule.

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	0,529505	2,289051	0,578161
Test granule 2	0,467887	2,022416	0,951529
Test granule 3	0,875659	0,695178	0,905991

3 CLUSTERS			
Test granule 1	0,665490	1,195455	**0,126463**
Test granule 2	1,470650	0,641749	**0,119320**
Test granule 3	0,891251	3,779527	**0,060955**
4 CLUSTERS			
Test granule 1	0,291799	1,516626	0,040361
Test granule 2	0,614654	1,119585	0,035267
Test granule 3	1,327124	1,884285	0,013563
5 CLUSTERS			
Test granule 1	1,250482	12,503896	0,024526
Test granule 2	2,682495	2,408998	0,015133
Test granule 3	1,944493	37,896157	0,015230

Table 166 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,537927	0,653439	0,578196
Test granule 2	0,478941	1,048399	0,951529
Test granule 3	0,884305	1,229083	0,905992
3 CLUSTERS			
Test granule 1	1,242014	0,826917	0,166191
Test granule 2	1,500004	0,392909	0,175287
Test granule 3	1,811944	6,123079	0,120113
4 CLUSTERS			
Test granule 1	0,724946	1,598969	0,266921
Test granule 2	1,561149	2,296811	0,268470
Test granule 3	1,701730	2,735338	0,268166
5 CLUSTERS			
Test granule 1	1,365507	13,735932	0,462938
Test granule 2	2,840253	2,920448	0,465003
Test granule 3	2,731959	39,274234	0,452989

Table 167 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the seasonalitytime series

This yields to an unexpected result, which is that the proposed approach to forecast the values of the first-phase granule is achieving lower values for error of type 3 than the approach proposed by the original article in all the Scenarios (See in Annex, Table 168 vs Table 170, and Table 169 vs Table 171). This is unexpected since the logic behind the proposed forecasting approach heavily pennalizes not forecasting the right second-phase granule, and since no model is correctly forecasting the second-phase granules we would expect the errors to be larger.

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	1,658919401	4,374534368	1,796685144
Test granule 2	2,388210802	4,739231626	2,362806236
Test granule 3	0,606289001	0,059963432	0,607935302
3 CLUSTERS			
Test granule 1	1,021308204	4,374456763	1,272904656
Test granule 2	2,034387528	2,618218263	1,575334076
Test granule 3	1,069066104	0,828939522	0,808466948
4 CLUSTERS			
Test granule 1	2,103359202	2,366064302	2,248015521
Test granule 2	1,241481069	1,778340757	2,571870824
Test granule 3	0,97569339	0,898092827	0,559229255
5 CLUSTERS			
Test granule 1	0,488713969	1,900543237	0,687849224
Test granule 2	1,102394209	0,743151448	0,998106904
Test granule 3	0,927392405	1,171690577	0,955150492

Table 168 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,561385809	4,380277162	0,351419069
Test granule 2	2,10096882	4,745	4,745
Test granule 3	0,674073136	0,059462729	0,059462729
3 CLUSTERS			
Test granule 1	0,304190687	4,381208426	0,166818182
Test granule 2	0,511202673	4,745935412	0,382616927
Test granule 3	1,318908579	1,082109705	1,108132208
4 CLUSTERS			
Test granule 1	4,400110865	0,114356984	4,470842572
Test granule 2	0,280979955	0,460077951	4,83596882
Test granule 3	1,170396624	1,088565401	0,080309423
5 CLUSTERS			
Test granule 1	0,587583149	0,191906874	0,493348115
Test granule 2	0,359020045	1,180634744	0,491091314
Test granule 3	1,266666667	1,265316456	1,111392405

Table 170 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	1,658658537	2,008625277	1,796718404
Test granule 2	2,38863029	2,3627951	2,362806236
Test granule 3	0,606253165	0,607935302	0,607935302
3 CLUSTERS			
Test granule 1	0,767926829	4,301507761	1,191973392
Test granule 2	1,5494098	1,233507795	1,542494432
Test granule 3	0,815108298	0,976658228	0,815144866
4 CLUSTERS			
Test granule 1	2,024988914	2,495731707	2,002184035
Test granule 2	2,759175947	2,420812918	2,356325167
Test granule 3	0,506104079	0,606098453	0,60956962
5 CLUSTERS			
Test granule 1	0,444911308	1,463325942	0,663447894
Test granule 2	1,163841871	0,768207127	1,011603563
Test granule 3	0,927443038	1,186739803	0,949248945

Table 169 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,561385809	0,351419069	0,351419069
Test granule 2	2,10096882	0,1022049	4,745
Test granule 3	0,674073136	1,206745429	0,059462729
3 CLUSTERS			
Test granule 1	0,304190687	4,381208426	0,944745011
Test granule 2	0,185389755	0,485634744	0,603741648
Test granule 3	1,194762307	1,082109705	1,357288326
4 CLUSTERS			
Test granule 1	0,305964523	4,352971175	4,470842572
Test granule 2	0,280979955	4,717572383	4,83596882
Test granule 3	1,170396624	0,106841069	0,080309423
5 CLUSTERS			
Test granule 1	0,587583149	4,520177384	0,54345898
Test granule 2	3,99986637	0,239643653	0,45233853
Test granule 3	0,194407876	1,265316456	1,246582278

Table 171 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the seasonality time series

What may be observed with this outcome is that the nature of forecasting approach proposed by the original article (weighted average of the membership strength) does not work well when having two (or more) groups that are completely different, situation that was mentioned before and can be observed in the shape of Figure 12 as opposite of Figure 10 and Figure 8.

The real values of the forecasted first-phase granules can be found in the Annex, Figure 48.

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 48 Test first-phase granules values and 3D representation of Scenario 3 applied to seasonality time series

The forecasted values for each combination of the number of clusters by the approach proposed in the original article can be found in the Annex, Table 136.

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	0,7291	3,8757	7,4213	Test granule 1	-0,8341	2,2941	5,8416
Test granule 2	1,3469	4,5229	8,0682	Test granule 2	-0,9744	2,1472	5,6937
Test granule 3	1,3469	4,5228	8,0682	Test granule 3	-0,9946	2,1275	5,6761
4 clusters	**a**	**m**	**b**	**5 clusters**	**a**	**m**	**b**
Test granule 1	2,0276	5,2728	8,7967	Test granule 1	-2,6998	0,4391	4,2851
Test granule 2	1,9651	5,1786	8,6717	Test granule 2	-2,8048	0,3286	4,1592
Test granule 3	1,929	5,1295	8,6109	Test granule 3	-2,8686	0,294	4,169

Table 136 Test fuzzy triangles with forecasting method 1 for Scenario 3 applied to seasonality time series

The forecasted values by the proposed approach for each combination of the number of clusters can be found in the Annex, Table 138.

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	-5,5589	-2,6938	0,9029	Test granule 1	-4,1497	-1,53	1,8356
Test granule 2	8,2581	11,765	15,307	Test granule 2	-4,1497	-1,53	1,8356
Test granule 3	8,2581	11,765	15,307	Test granule 3	-4,1497	-1,53	1,8356
4 clusters	**a**	**m**	**b**	**5 clusters**	**a**	**m**	**b**
Test granule 1	10,366	12,2	13,581	Test granule 1	-3,144	-1,121	0,305
Test granule 2	10,366	12,2	13,581	Test granule 2	-3,144	-1,121	0,305
Test granule 3	10,366	12,2	13,581	Test granule 3	-3,144	-1,121	0,305

Table 138 Test fuzzy triangles with forecasting method 2 for Scenario 3 applied to seasonality time series

It can be observed that the forecast with the lowest error is achieved by the model with 3 clusters forecasted with the method 2 (**proposed approach**).

As it can be observed, even we are dealing with a lower length period when comparing with the stationary time series case study, the average error of type 3 for Scenario 3, which is the lowest among all, is 0,552, which is very high.

There is an important point to be analyzed here, which is that the Principle of Justifiable Granularization is done only with 4 points. The discussion here should be on the number of points that should be used to optimize each granule, perhaps if we used period of 4 months but with weekly data the information could reflect better the reality of the time series.

4.5. Summary of the output of the case studies

The global conclussions for the different studies:

- **Case study 1, trend time series:** The algorithm has worked quite well, yielding to a low error on both the error of type 2 and type 3. The most important improvement when analying trend time series would be, as stated before, a forecasting approach that can include in the forecasted values the increase or decrease caused by an upward or downward trend, respectively.
- **Case study 2, stationary time series:** The algorithm seems to still have room for improvement. A key point for improvement here is the unacurracy on the forecast of the second-phase granules. Even Formula 18 has had low errors of type 2, it has failed to correctly forecast all the second-phase granules correctly regardless of the number of cluster selection, with exception of S1 and S3 with two clusters, which could even be a casualty taking into account the poor metrics of error of type 2 and 3 for these models. The forecasting approach of the second-phase granules should be reviewed and computed with a different technique. It must be highlited that this could be either an issue of the forecasting technique or the way the FCM is built, which ultimately may entail to consider an optimization with different a objective function and different constraints.
- **Case study 3, seasonal time series:** The algorithm seems to still have room for improvement. The key points extracted from the analysis of the case study 2 results can also be applied for this case study. Another key point here is the number of points per period. In the case study 2 the length of each period was discussed, but the number of values per period seems to be also an important feature. A research that could be considered at this point is performing the granularization and analysis with FCMs of the same time series but with different frequencies. In this case monthly temperatures were considered, so extending with the daily and weekly series of the same information would be recommended.

5. Conclusions

The goal of this study was computing different approaches to time series that display different behaviour and properties and assess which is the best choice for each type of time series. The overall conclusions on the different approaches of each step of the methodology are:

- **Optimization algorithm:** Two different optimization algorithms have been used: the genetic algorithm optimization was proposed in the original article and IPOPT algorithm optimization has been proposed in this article. The optimization done by the genetic algorithm has yielded better results, the IPOPT optimization algorithm does not seem as the most adecuate optimization algorithm to be used for fuzzy cognitive maps. This can be seen in the different case studies, where the error of type 2 has mostly been lower for the optimization with genetic algorithm.
- **Membership degree function:** Three different functions have been proposed, Eqs. 16 and 17 were proposed by the original article, and Eq. 18 was proposed in this article. For all the case studies, the proposed Eq. 18 has yielded to a lower error of type 2, so this function should be the first choice when measuring the degree of membership of each first-phase granule with each second-phase granule.
- **Forecasting approach:** Two different approaches for the actual values of the first-phase granules have been tested. Eqs. 27, 28 and 29 were proposed by the original article, and Eqs. 30, 31 and 31 have been proposed in this article. The proposed equations have generally yielded to a lower error of type 3, with exception of the stationary time series. Although it is considered that other forecasting techniques should be explored so implicit properties of the time series should be included, such as the increasing (or decreasing) trend.

The thoughts on this study is that fuzzy cognitive maps in the modeling of granular time is a technique that should be explored more in depth before being applied. It is a very sensitive process to decissions such as the length of the periods or the amount of information that is contained in each period, so this is a key point that needs to be explored. Also, the objective function of the optimization and the forecasting methodologies should be further studied. For instance, fuzzy cognitive maps suggest that the activation of a cluster with itself should be 0, which is a constraint of the model, but in the trend time series it may not be the case.

Although some of the results can be improved, the functions proposed to measure the degree of membership and the approaches to forecast the values of the first-phase granules have improved the results of the original article, which might introduce another base point to further explore this methodology.

limitacionesThere are several paths to extend this study and investigate deeper the many different casuistics. For example, in a further extension of this study, traditional models such as ARIMA (and its variations) could be used to forecast the time series and analyze which model yields to more accurate forecasted values (considering only the minimum, median and maximum values). Also, with respect to the case study 1:

- Before performing the granularization, performing a detrending process to the raw time series and add the effect of the trend to the forecast values of the first-phase granules for xxx.

With respect to case study 2:

- Try the next modifications in the FCM:
    - Use a formula that penalyses more the error displayed in Equation 22, to xx.
    - Carry out modifications to the Equation 20, to see how different transformations affect the FCM relations.
    - Remove the constraint of Equation 23, which would mean that a center can have an activation relationship with itself.
- Carry an study with a specific behaviour time series (such as trend, seasonal, stationary…) with different lengths of periods, to xxx.

With respect to case study 3:

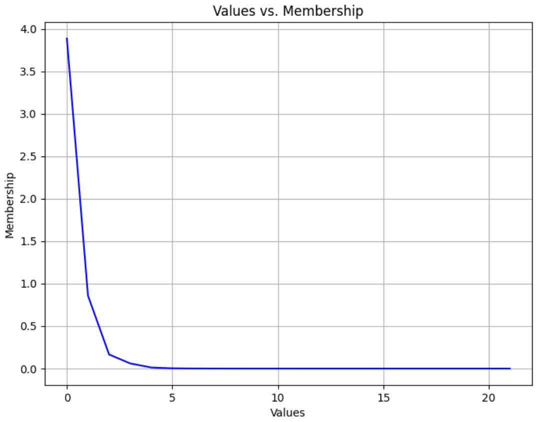
- Carry out an study of the same time series with different frequencies but same intervales. For instance, in this case study quarterly periods were selected with monthly data. It would be interesting to see how the model works with quarterly periods with weekly data and daily data.
- Try another ways of forecasting the second-phase ganules. Alternative options might yield to better forecasts of the membership values.

6. Annex

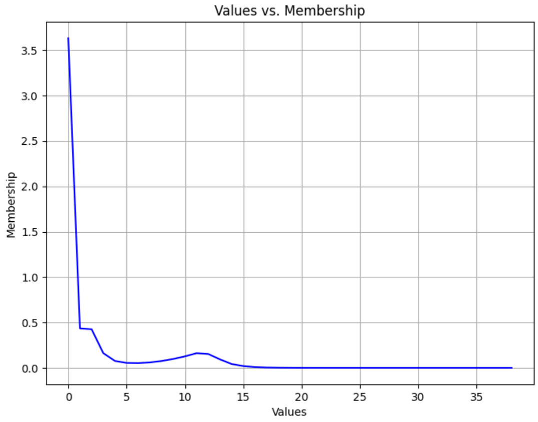
6.1. Time series with trend

6.1.1. Errors of Fuzzy C-means

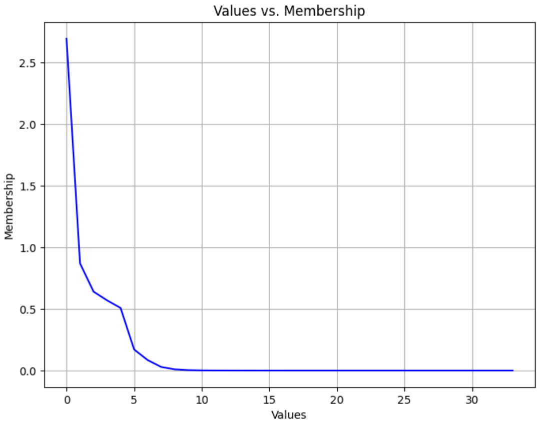
2 Clusters



3 Clusters



4 Clusters



5 Clusters

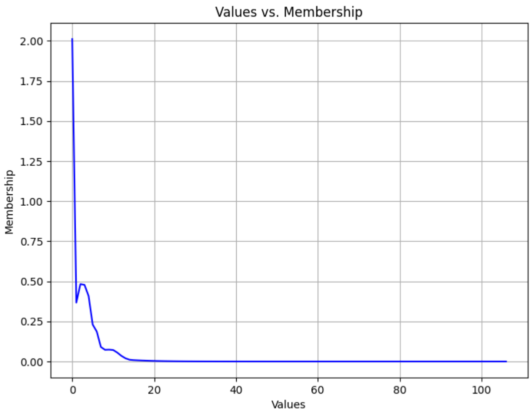


Figure 13 Trend time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

6.1.2. First-phase granules (blue) and second-phase granules (red)

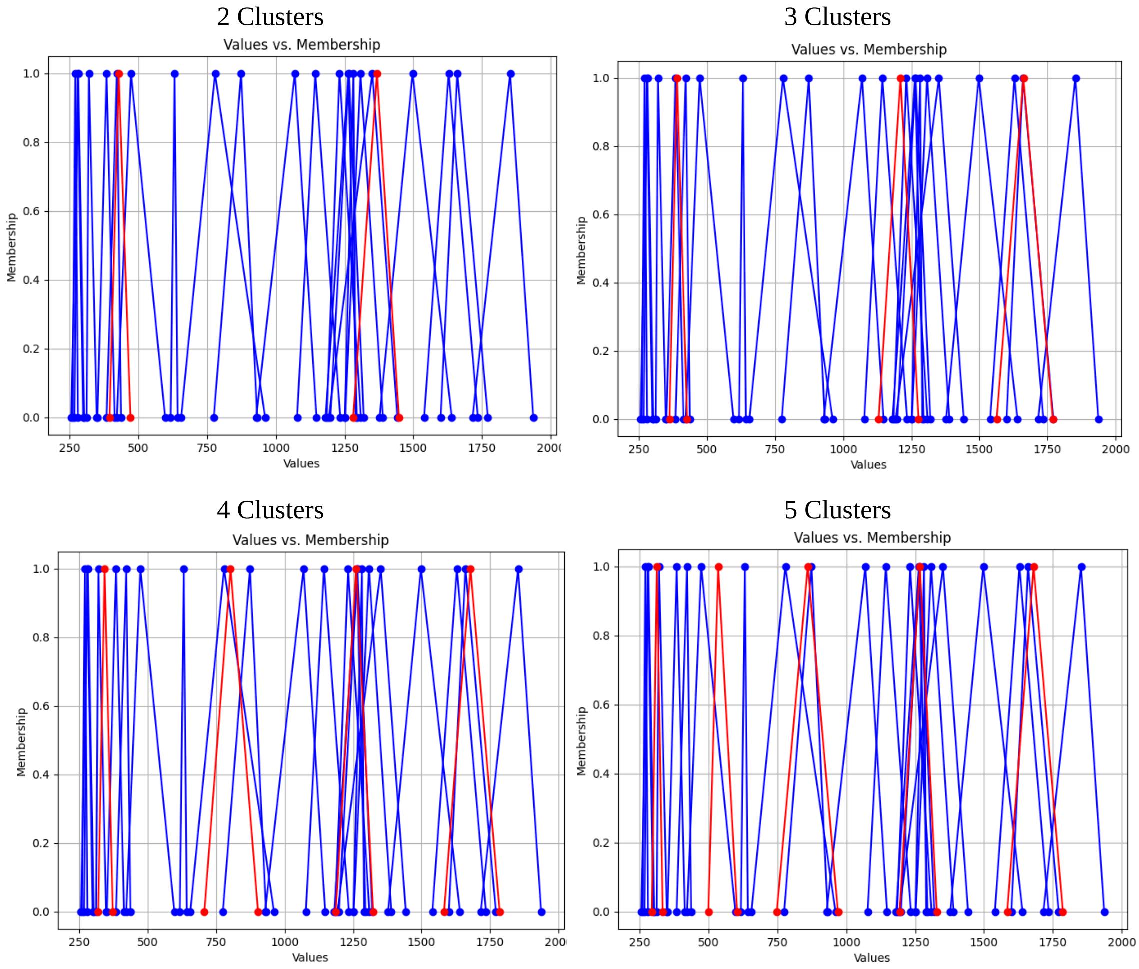


Figure 14 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the trend time series

6.1.3. Scenario 1 – Optimization with genetic algorithm

6.1.3.i First-phase granules clusterization

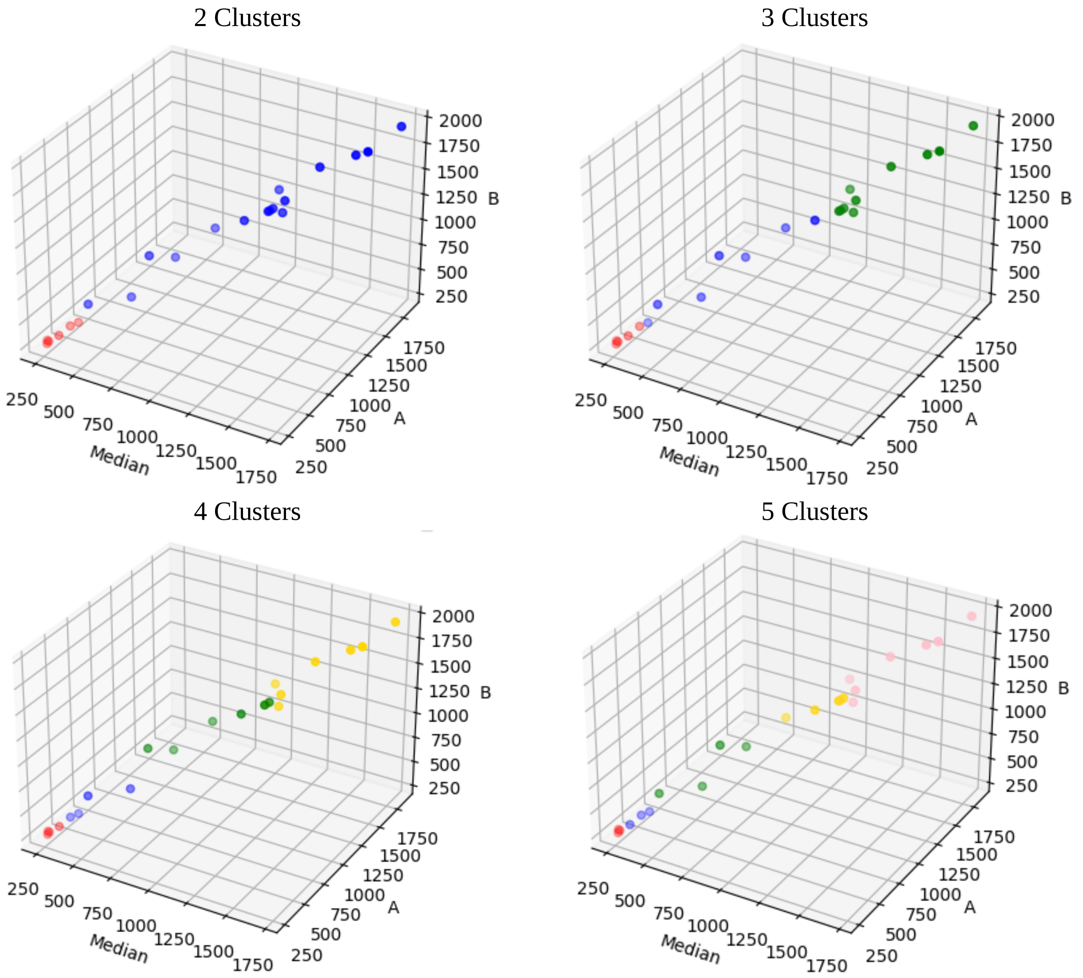


Figure 15 First-phase granules of trend time series by applying Scenario 1

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.3.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	0,003877
Cluster 1	3,938447	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	-1,302203	-0,76729
Cluster 1	-1,325703	0	-0,183314
Cluster 2	-0,114881	-0,460023	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,124377	-3,136749	-1,367578
Cluster 1	-1,310937	0	0,593793	-3,130197
Cluster 2	-2,90847	-0,454137	0	-0,908183
Cluster 3	1,515213	-1,846856	-1,161674	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-1,18486	-3,002369	-1,949433	-0,709973
Cluster 1	-0,540841	0	-2,414471	-1,216802	-2,192503
Cluster 2	1,461223	-3,201162	0	-3,438551	0,613098
Cluster 3	2,067465	-0,582906	-2,876586	0	-1,053311
Cluster 4	-0,83338	0,678636	-0,103868	-3,083061	0

Table 4 Fuzzy cognitive map weights for Scenario 1 of trend time series

6.1.3.iii Test membership values

Cluster 0	Cluster 1
0,245575	0,754425
0,245575	0,754425
0,245575	0,754425

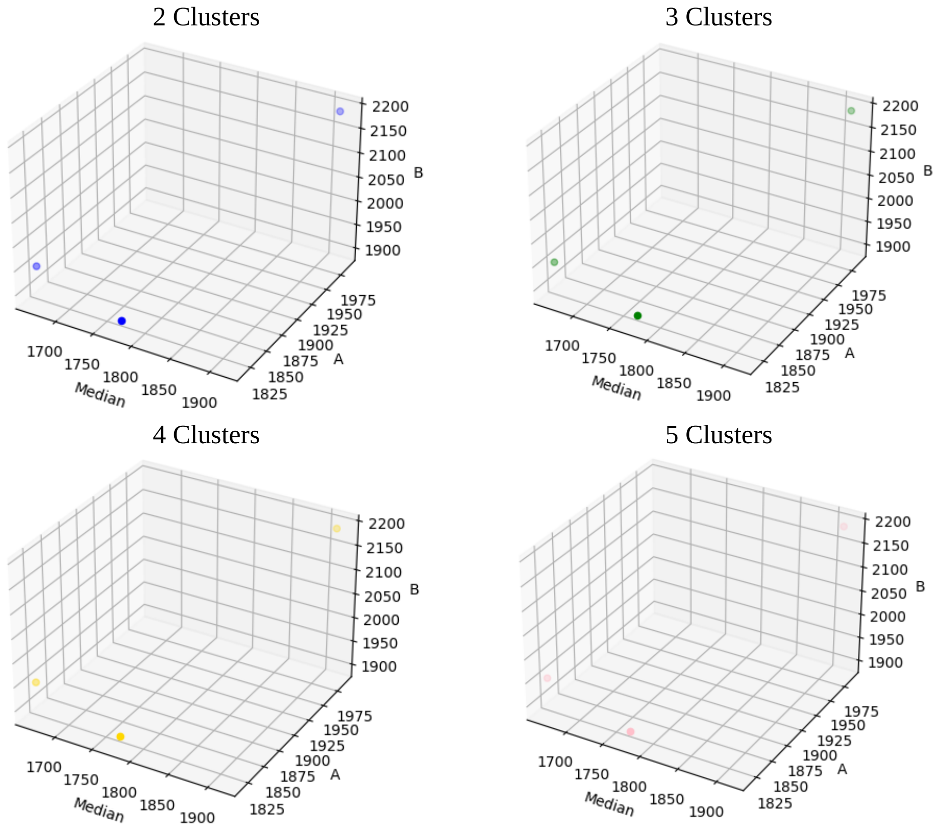
Cluster 0	Cluster 1	Cluster 2
0,122923	0,367547	0,509530
0,122923	0,367547	0,509530
0,122923	0,367547	0,509530

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,084971	0,205977	0,301850	0,407202
0,084971	0,205977	0,301850	0,407202
0,084971	0,205977	0,301850	0,407202

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,066386	0,120239	0,193079	0,264562	0,355734
0,066386	0,120239	0,193079	0,264562	0,355734
0,066386	0,120239	0,193079	0,264562	0,355734

Table 5 Membership degree between first and second-phase granules (clusters) for Scenario 1 of trend time series

6.1.3.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 16 Test first-phase granules values and 3D representation of pr ocess 1 applied to trend time series

6.1.3.v Test membership forecast

Cluster 0	Cluster 1
0,500731	0,724558
0,500702	0,877837
0,500851	0,877825

Cluster 0	Cluster 1	Cluster 2
0,295342	0,436258	0,454327
0,285631	0,383476	0,441613
0,301912	0,387077	0,447885

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,185716	0,230272	0,329490	0,353815
0,184108	0,239518	0,275668	0,371301
0,207068	0,224452	0,272603	0,381396

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,183852	0,167430	0,272989	0,296725	0,308009
0,140021	0,142470	0,249968	0,304238	0,272342
0,153716	0,161572	0,244044	0,310138	0,272108

Table 6 Test membership degree forecast values for Scenario 1 applied to trend time series

6.1.3.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,065997	0,037497	0,014354	0,025723
0,080320	0,031341	0,012928	0,017681
0,080393	0,036219	0,016770	0,021003

Table 7 Error of type 2(membership degree forecast) values for Scenario 1 applied to trend time series

6.1.3.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	919,22	984,19	1048,83	Test granule 1	1104,72	1178,84	1253,95
Test granule 2	959,50	1026,89	1093,21	Test granule 2	1104,77	1178,72	1254,20
Test granule 3	959,43	1026,83	1093,14	Test granule 3	1096,74	1170,18	1245,27

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	1066,84	1144,29	1223,33	Test granule 1	963,34	1037,76	1116,14
Test granule 2	1067,26	1145,40	1226,00	Test granule 2	986,31	1062,32	1141,52
Test granule 3	1060,89	1137,81	1217,35	Test granule 3	972,04	1046,48	1124,55

Table 8 Test fuzzy triangles with forecasting method 1 for Scenario 1 applied to trend time series

6.1.3.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,462087	0,355447	0,37422	0,432022
0,433596	0,349345	0,36756	0,413265
0,494646	0,423622	0,43940	0,484198

Table 9 Error of type 3 for method 1 for Scenario 1 applied to trend time series

6.1.3.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	1132,4	1216,6	1299,7	Test granule 1	1368,8	1454,9	1536,4
Test granule 2	1132,4	1216,6	1299,7	Test granule 2	1368,8	1454,9	1536,4
Test granule 3	1132,4	1216,6	1299,7	Test granule 3	1368,8	1454,9	1536,4

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	1410,2	1502,6	1587,4	Test granule 1	1410,5	1502,8	1587,5
Test granule 2	1410,2	1502,6	1587,4	Test granule 2	1116,6	1195,6	1268,2
Test granule 3	1410,2	1502,6	1587,4	Test granule 3	1116,6	1195,6	1268,2

Table 10 Test fuzzy triangles with forecasting method 2 for Scenario 1 applied to trend time series

6.1.3.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,335196	0,205568	0,180037	0,179926
0,328935	0,198087	0,172315	0,341483
0,401227	0,284474	0,261479	0,412423

Table 11 Error of type 3 for method 2 for Scenario 1 applied to trend time series

6.1.4. Scenario 2– Optimization with genetic algorithm

6.1.4.i First-phase granules clusterization

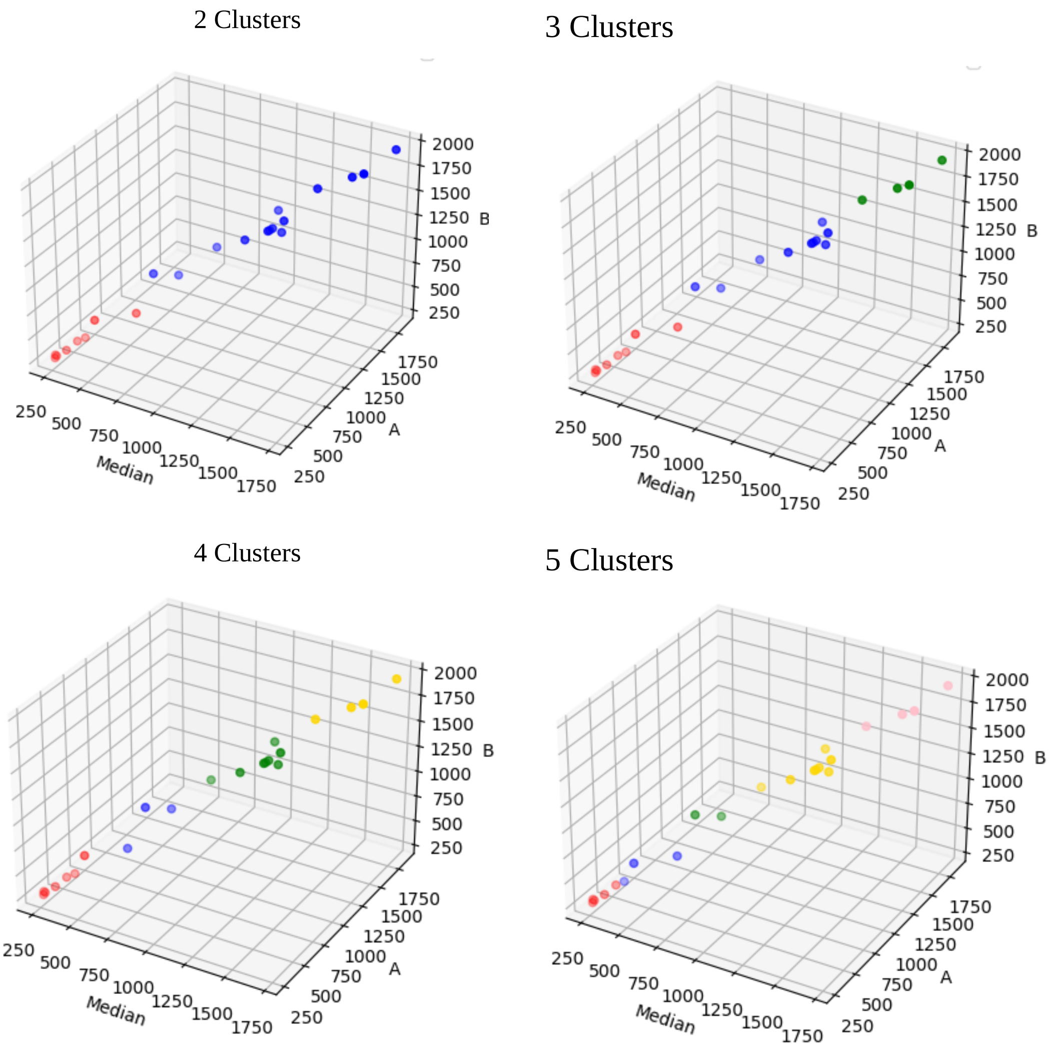


Figure 17 First-phase granules of trend time series by applying Scenario 2

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.4.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	4,778545
Cluster 1	3,758014	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	1,373397	1,176782
Cluster 1	-0,970614	0	0,119083
Cluster 2	-0,304046	0,062201	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,492031	0,522298	-2,799093
Cluster 1	-0,006851	0	-1,02118	-1,5246
Cluster 2	-0,875064	-1,032026	0	-0,607885
Cluster 3	0,388677	-0,846714	-0,93471	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-0,690373	-1,875638	0,757646	-0,642861
Cluster 1	-0,377972	0	-1,89897	-3,354502	1,759064
Cluster 2	-0,06028	0,292486	0	-4,352091	-0,299248
Cluster 3	-1,642231	0,732846	-2,422768	0	-0,822131
Cluster 4	0,199307	-1,70643	0,643982	-2,164981	0

Table 12 Fuzzy cognitive map weights for Scenario 2 of trend time series

6.1.4.iii Test membership values

Cluster 0	Cluster 1
0,240173	0,759827
0,240173	0,759827
0,240173	0,759827

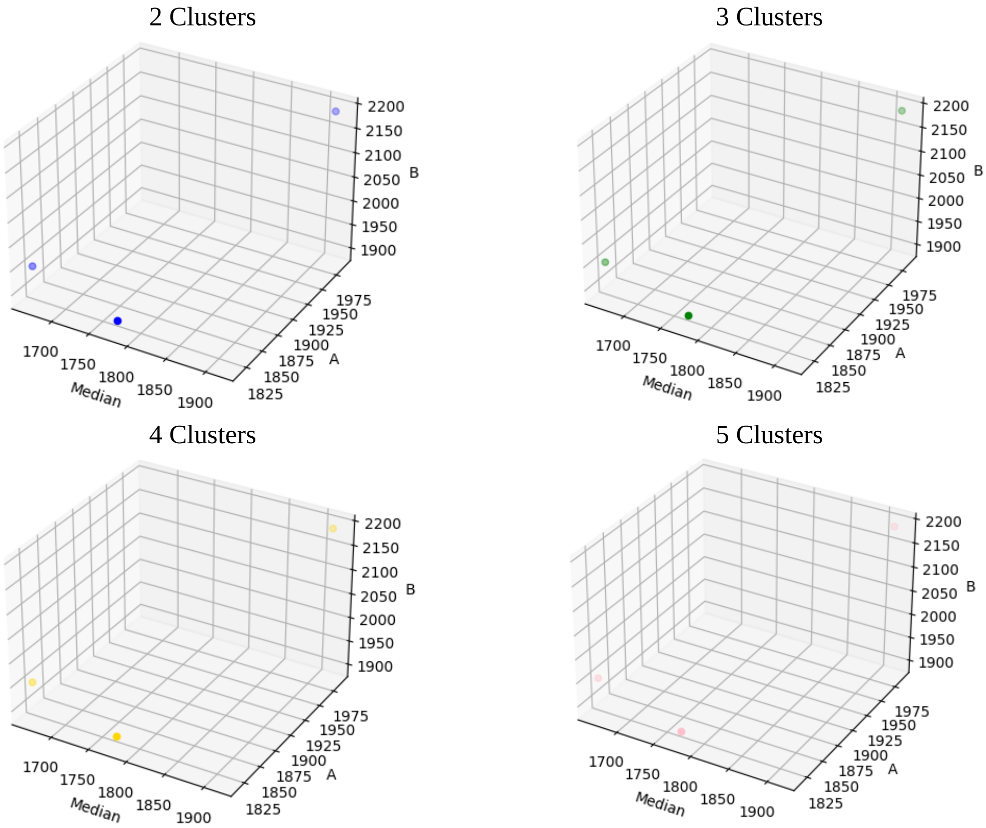
Cluster 0	Cluster 1	Cluster 2
0,120429	0,369418	0,510153
0,120429	0,369418	0,510153
0,120429	0,369418	0,510153

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,084355	0,196725	0,307474	0,411447
0,084355	0,196725	0,307474	0,411447
0,084355	0,196725	0,307474	0,411447

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,067136	0,11703	0,184172	0,270759	0,360903
0,067136	0,11703	0,184172	0,270759	0,360903
0,067136	0,11703	0,184172	0,270759	0,360903

Table 13 Membership degree between first and second-phase granules (clusters) for Scenario 2 of trend time series

6.1.4.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 18 Test first-phase granules values and 3D representation of pr ocess 2 applied to trend time series

6.1.4.v Test membership forecast

Cluster 0	Cluster 1
0,974191	0,711478
0,967699	0,974938
0,990611	0,974335

Cluster 0	Cluster 1	Cluster 2
0,751699	0,485969	0,496591
0,777613	0,338391	0,450581
0,730071	0,331567	0,446362

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,290225	0,280526	0,371228	0,396234
0,314929	0,271866	0,313385	0,384211
0,314752	0,28742	0,312231	0,401141

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,388623	0,343323	0,221631	0,317041	0,342109
0,346956	0,263184	0,196996	0,230618	0,258829
0,367524	0,305013	0,264043	0,255979	0,320304

Table 14 Test membership degree forecast values for Scenario 2 applied to trend time series

6.1.4.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,541119	0,41227	0,053701	0,158461
0,575567	0,436401	0,059587	0,111855
0,609171	0,377164	0,061437	0,133817

Table 15 Error of type 2(membership degree forecast) values for Scenario 2 applied to trend time series

6.1.4.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters

	a	m	b
Test granule 1	769,21	825,19	883,52
Test granule 2	839,92	900,14	961,45
Test granule 3	834,6	894,5	955,58

3 clusters

	a	m	b
Test granule 1	921,21	983,92	1049,6
Test granule 2	873,44	932,87	996,71
Test granule 3	886,47	946,69	1011,2

4 clusters

	a	m	b
Test granule 1	1015	1089,4	1165,8
Test granule 2	990,67	1063,8	1139,4
Test granule 3	994,84	1068,5	1144,9

5 clusters

	a	m	b
Test granule 1	850,63	912,93	982,41
Test granule 2	822,2	883,69	952,15
Test granule 3	839,98	904,04	975,12

Table 16 Test fuzzy triangles with forecasting method 1 for Scenario 2 applied to trend time series

6.1.4.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,54851	0,461643	0,404146	0,49967
0,503136	0,484465	0,412584	0,511131
0,559423	0,53321	0,473504	0,553768

Table 17 Error of type 3 for method 1 for Scenario 2 applied to trend time series

6.1.4.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters

	a	m	b
Test granule 1	362,89	387,92	421,49
Test granule 2	1214,1	1305,1	1390,3
Test granule 3	362,89	387,92	421,49

3 clusters

	a	m	b
Test granule 1	359,22	383,62	416,56
Test granule 2	359,22	383,62	416,56
Test granule 3	359,22	383,62	416,56

4 clusters

	a	m	b
Test granule 1	1562,9	1663,6	1773,8
Test granule 2	1562,9	1663,6	1773,8
Test granule 3	1562,9	1663,6	1773,8

5 clusters

	a	m	b
Test granule 1	290,16	307,97	329,68
Test granule 2	290,16	307,97	329,68
Test granule 3	290,16	307,97	329,68

Table 18 Test fuzzy triangles with forecasting method 2 for Scenario 2 applied to trend time series

6.1.4.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,786401	0,788752	0,088921	0,830948
0,280965	0,786763	0,080341	0,829356
0,807617	0,809734	0,179413	0,847739

Table 19 Error of type 3 for method 2 for Scenario 2 applied to trend time series

6.1.5. Scenario 3– Optimization with genetic algorithm

6.1.5.i First-phase granules clusterization

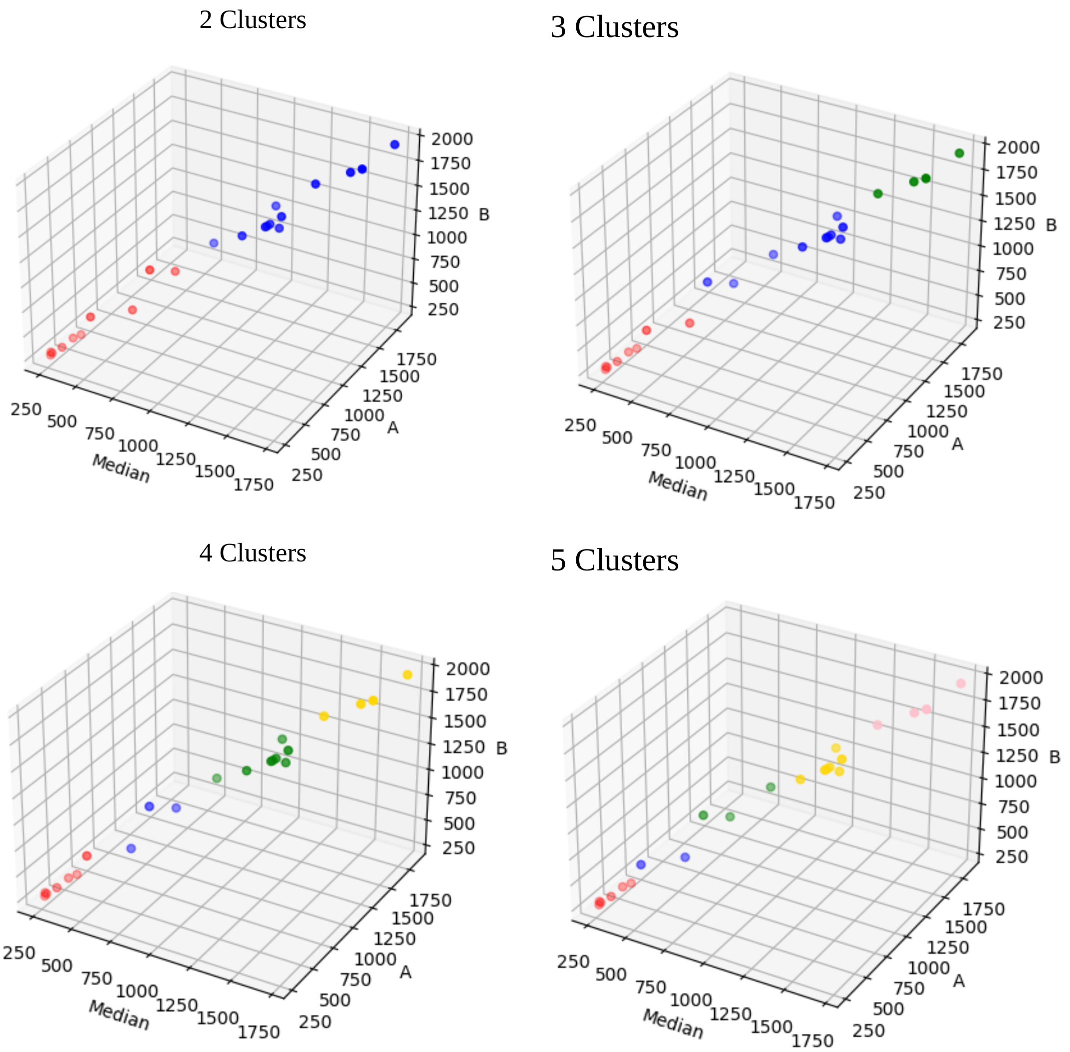


Figure 19 First-phase granules of trend time series by applying Scenario 3

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.5.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	1,841428
Cluster 1	17,042032	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	-0,401491	0,155415
Cluster 1	-0,791052	0	0,149077
Cluster 2	-0,412135	-0,240369	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,036887	-1,177375	-2,752675
Cluster 1	-1,861245	0	-0,117061	-1,378144
Cluster 2	-3,551229	-0,525632	0	-0,037563
Cluster 3	-0,604339	-0,92955	-1,502347	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-1,576916	-0,43198	-1,762555	-2,613527
Cluster 1	-1,020869	0	-1,066803	-3,009361	-1,01019
Cluster 2	-1,410951	0,20296	0	-3,166889	-1,556052
Cluster 3	-0,399344	0,390643	-3,630084	0	-1,719461
Cluster 4	-1,085245	-2,536664	1,815153	-3,806537	0

Table 20 Fuzzy cognitive map weights for Scenario 3 of trend time series

6.1.5.iii Test membership values

Cluster 0	Cluster 1
0,249266	0,750734
0,245215	0,754785
0,294176	0,705824

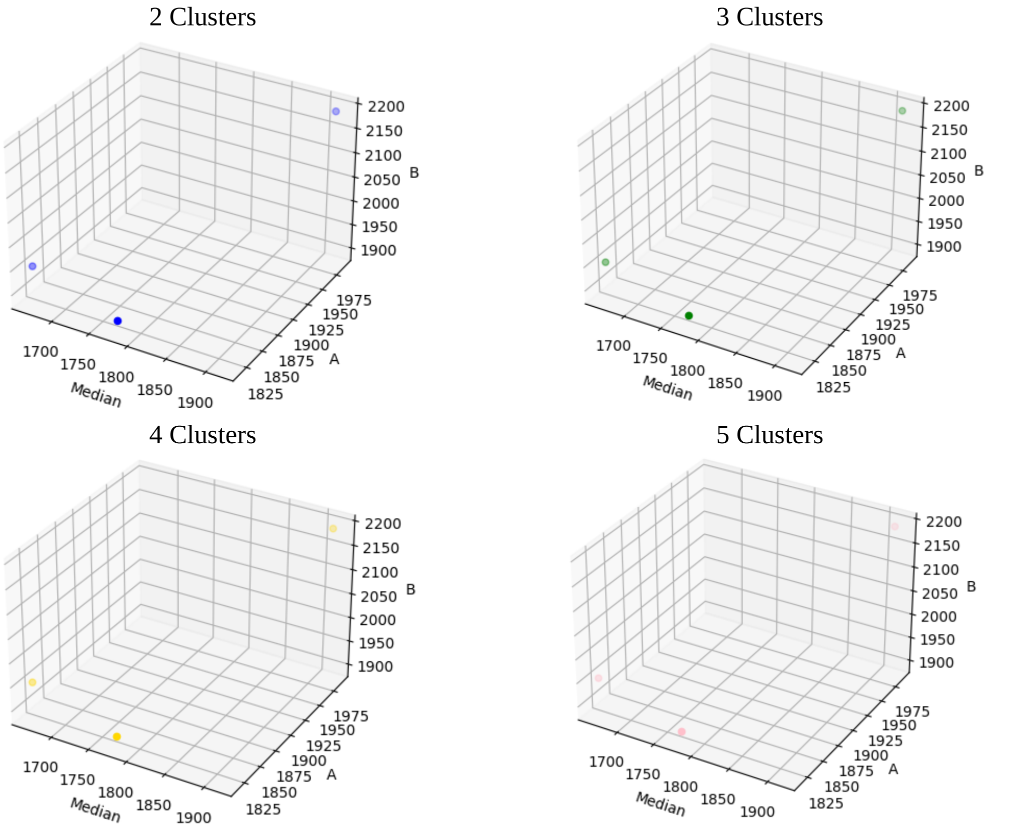
Cluster 0	Cluster 1	Cluster 2
0,177766	0,360023	0,462211
0,174311	0,360405	0,465284
0,210848	0,354084	0,435068

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,180320	0,227632	0,274324	0,317725
0,178876	0,227377	0,274630	0,319117
0,194157	0,232165	0,269310	0,304369

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,165503	0,178493	0,195894	0,218447	0,241664
0,164842	0,178102	0,195962	0,218671	0,242422
0,171790	0,182422	0,196711	0,214971	0,234106

Table 21 Membership degree between first and second-phase granules (clusters) for Scenario 3 of trend time series

6.1.5.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 20 Test first-phase granules values and 3D representation of Scenario 3 applied to trend time series

6.1.5.v Test membership forecast

Cluster 0	Cluster 1
0,798083	0,986909
0,860244	0,999999
0,863117	1,000000

Cluster 0	Cluster 1	Cluster 2
0,481830	0,481413	0,459880
0,469585	0,422482	0,422067
0,474017	0,423470	0,426760

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,234069	0,308676	0,315082	0,324553
0,222186	0,284994	0,267828	0,288691
0,249835	0,300955	0,278949	0,309692

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,200848	0,217841	0,220360	0,245532	0,248258
0,179417	0,193129	0,197333	0,227601	0,213292
0,206133	0,215164	0,219834	0,253641	0,232758

Table 22 Test membership degree forecast values for Scenario 3 applied to trend time series

6.1.5.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,356979	0,107196	0,011165	0,004173
0,438391	0,092908	0,006167	0,001369
0,410233	0,074141	0,007953	0,004283

Table 23 Error of type 2(membership degree forecast) values for Scenario 3 applied to trend time series

6.1.5.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	885,15	948,08	1011,3	Test granule 1	1010	1078,3	1148,3
Test granule 2	871,56	933,68	996,31	Test granule 2	994,74	1062,1	1131,4
Test granule 3	870,83	932,9	995,5	Test granule 3	994,74	1062	1131,4
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	998,19	1074,2	1152,2	Test granule 1	899,5	968,48	1043,4
Test granule 2	984,21	1059,8	1137,5	Test granule 2	896,67	965,53	1040,1
Test granule 3	977,18	1052,1	1129,4	Test granule 3	890,38	958,85	1033

Table 24 Test fuzzy triangles with forecasting method 1 for Scenario 3 applied to trend time series

6.1.5.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,481716	0,410265	0,412465	0,469527
0,484736	0,413624	0,414865	0,466201
0,540625	0,476793	0,481633	0,527003

Table 25 Error of type 3 for method 1 for Scenario 3 applied to trend time series

6.1.5.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	1290,8	1378,8	1458,6	Test granule 1	359,22	383,62	416,56
Test granule 2	1290,8	1378,8	1458,6	Test granule 2	359,22	383,62	416,56
Test granule 3	1290,8	1378,8	1458,6	Test granule 3	359,22	383,62	416,56
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	1562,9	1663,6	1773,8	Test granule 1	1563,4	1663,9	1774,1
Test granule 2	1562,9	1663,6	1773,8	Test granule 2	1191,5	1264,3	1326,3
Test granule 3	1562,9	1663,6	1773,8	Test granule 3	1191,5	1264,3	1326,3

Table 26 Test fuzzy triangles with forecasting method 2 for Scenario 3 applied to trend time series

6.1.5.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,247821	0,788751	0,08892	0,088719
0,240738	0,786762	0,08034	0,304393
0,322531	0,809734	0,179412	0,379328

Table 27 Error of type 3 for method 2 for Scenario 3 applied to trend time series

6.1.6. Scenario 4– Optimization with IPOPT algorithm

6.1.6.i First-phase granules clusterization

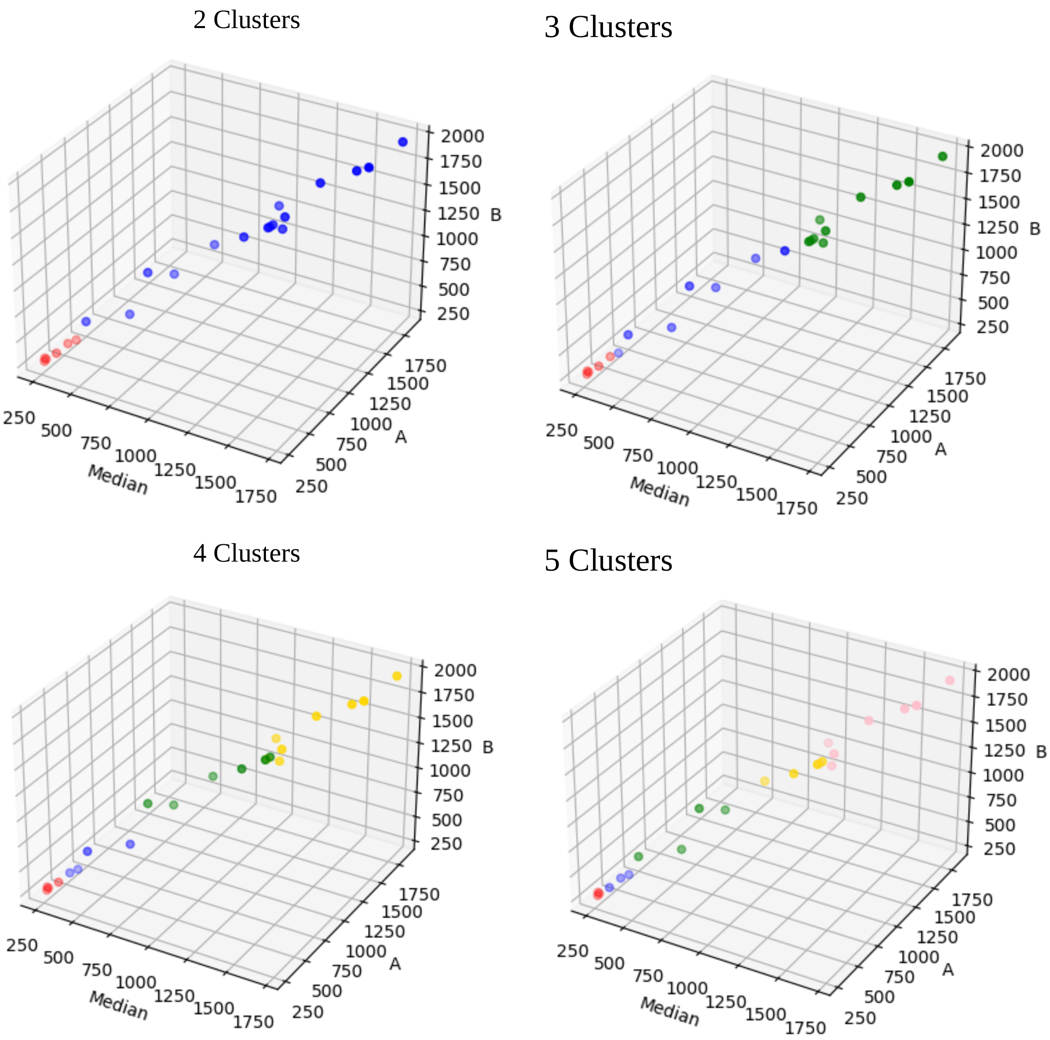


Figure 21 First-phase granules of trend time series by applying Scenario 4

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.6.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	0,000018
Cluster 1	36,434299	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	0,000022	0,000022
Cluster 1	0,000024	0	0,000013
Cluster 2	0,000082	0,000028	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,000036	0,000036	0,000036
Cluster 1	0,000047	0	0,000034	0,000034
Cluster 2	0,000058	0,000024	0	0,000019
Cluster 3	0,000058	0,000024	0,000016	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,000061	0,000061	0,000061	0,000061
Cluster 1	0,000071	0	0,000059	0,000059	0,000059
Cluster 2	0,000089	0,000049	0	0,000038	0,000038
Cluster 3	0,000121	0,000067	0,000041	0	0,000032
Cluster 4	0,000074	0,000041	0,000026	0,000019	0

Table 28 Fuzzy cognitive map weights for Scenario 4 of trend time series

6.1.6.iii Test membership values

Cluster 0	Cluster 1
0,245575	0,754425
0,245575	0,754425
0,245575	0,754425

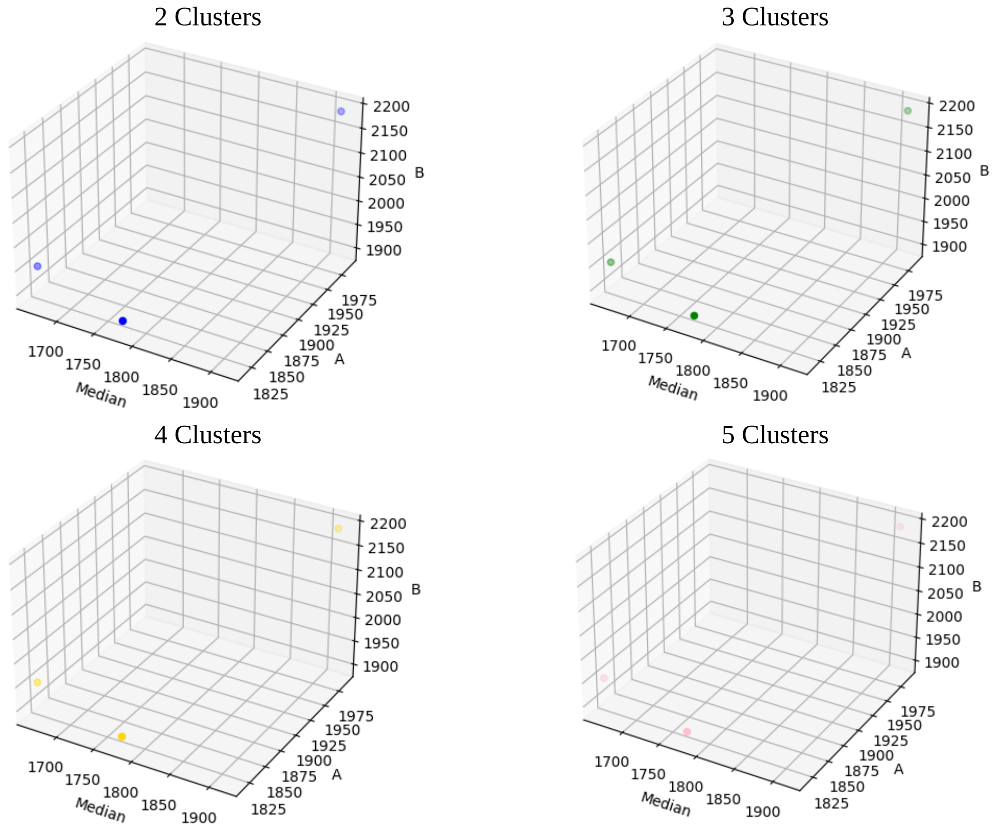
Cluster 0	Cluster 1	Cluster 2
0,122923	0,367547	0,509530
0,122923	0,367547	0,509530
0,122923	0,367547	0,509530

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,084971	0,205977	0,301850	0,407202
0,084971	0,205977	0,301850	0,407202
0,084971	0,205977	0,301850	0,407202

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,066386	0,120239	0,193079	0,264562	0,355734
0,066386	0,120239	0,193079	0,264562	0,355734
0,066386	0,120239	0,193079	0,264562	0,355734

Table 29 Membership degree between first and second-phase granules (clusters) for Scenario 4 of trend time series

6.1.6.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 22 Test first-phase granules values and 3D representation of pr ocess 4 applied to trend time series

6.1.6.v Test membership forecast

Cluster 0	Cluster 1
0,5000034124	0,9998699378
0,5000045226	0,9999999877
0,5000045232	0,9999999877

Cluster 0	Cluster 1	Cluster 2
0,5000047872	0,5000024313	0,5000050681
0,5000054582	0,5000046984	0,5000137557
0,5000054582	0,5000046984	0,5000137557

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,5000083292	0,5000070006	0,5000043873	0,5000036726
0,5000136542	0,5000143464	0,5000126006	0,5000122044
0,5000136544	0,5000143466	0,5000126008	0,5000122045

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,5000141698	0,5000130954	0,5000089010	0,5000088509	0,5000049360
0,5000303552	0,5000308300	0,5000268991	0,5000325719	0,5000199553
0,5000303563	0,5000308311	0,5000269001	0,5000325730	0,5000199560

Table 30 Test membership degree forecast values for Scenario 4 applied to trend time series

6.1.6.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,124977	0,159826	0,306587	0,502717
0,125041	0,159827	0,306600	0,502771
0,125041	0,159827	0,306600	0,502771

Table 31 Error of type 2(membership degree forecast) values for Scenario 4 applied to trend time series

6.1.6.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters

	a	m	b
Test granule 1	985,95	1054,9	1122,4
Test granule 2	985,98	1055	1122,4
Test granule 3	985,98	1055	1122,4

3 clusters

	a	m	b
Test granule 1	1018,4	1087,2	1157,8
Test granule 2	1018,4	1087,2	1157,8
Test granule 3	1018,4	1087,2	1157,8

4 clusters

	a	m	b
Test granule 1	948,68	1021,3	1096
Test granule 2	948,68	1021,3	1096
Test granule 3	948,68	1021,3	1096

5 clusters

	a	m	b
Test granule 1	863,79	931,01	1004,5
Test granule 2	863,79	931,01	1004,5
Test granule 3	863,79	931,01	1004,5

Table 32 Test fuzzy triangles with forecasting method 1 for Scenario 4 applied to trend time series

6.1.6.viii Error type (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,42364	0,405381	0,441362	0,489951
0,418198	0,39978	0,436101	0,485148
0,480874	0,46444	0,496848	0,540611

Table 33 Error of type 3 for method 1 for Scenario 4 applied to trend time series

6.1.6.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters

	a	m	b
Test granule 1	1132,4	1216,6	1299,7
Test granule 2	1132,4	1216,6	1299,7
Test granule 3	1132,4	1216,6	1299,7

3 clusters

	a	m	b
Test granule 1	1368,8	1454,9	1536,4
Test granule 2	1368,8	1454,9	1536,4
Test granule 3	1368,8	1454,9	1536,4

4 clusters

	a	m	b
Test granule 1	283,04	298,77	320,68
Test granule 2	496,36	542,83	598,84
Test granule 3	496,36	542,83	598,84

5 clusters

	a	m	b
Test granule 1	269,55	285,81	303,97
Test granule 2	1116,6	1195,6	1268,2
Test granule 3	1116,6	1195,6	1268,2

Table 34 Test fuzzy triangles with forecasting method 2 for Scenario 4 applied to trend time series

6.1.6.x Error type (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,335188	0,205567	0,835562	0,843426
0,328928	0,198086	0,698732	0,34149
0,40122	0,284474	0,731187	0,412429

Table 35 Error of type 3 for method 2 for Scenario 4 applied to trend time series

6.1.7. Scenario 5– Optimization with IPOPT algorithm

6.1.7.i First-phase granules clusterization

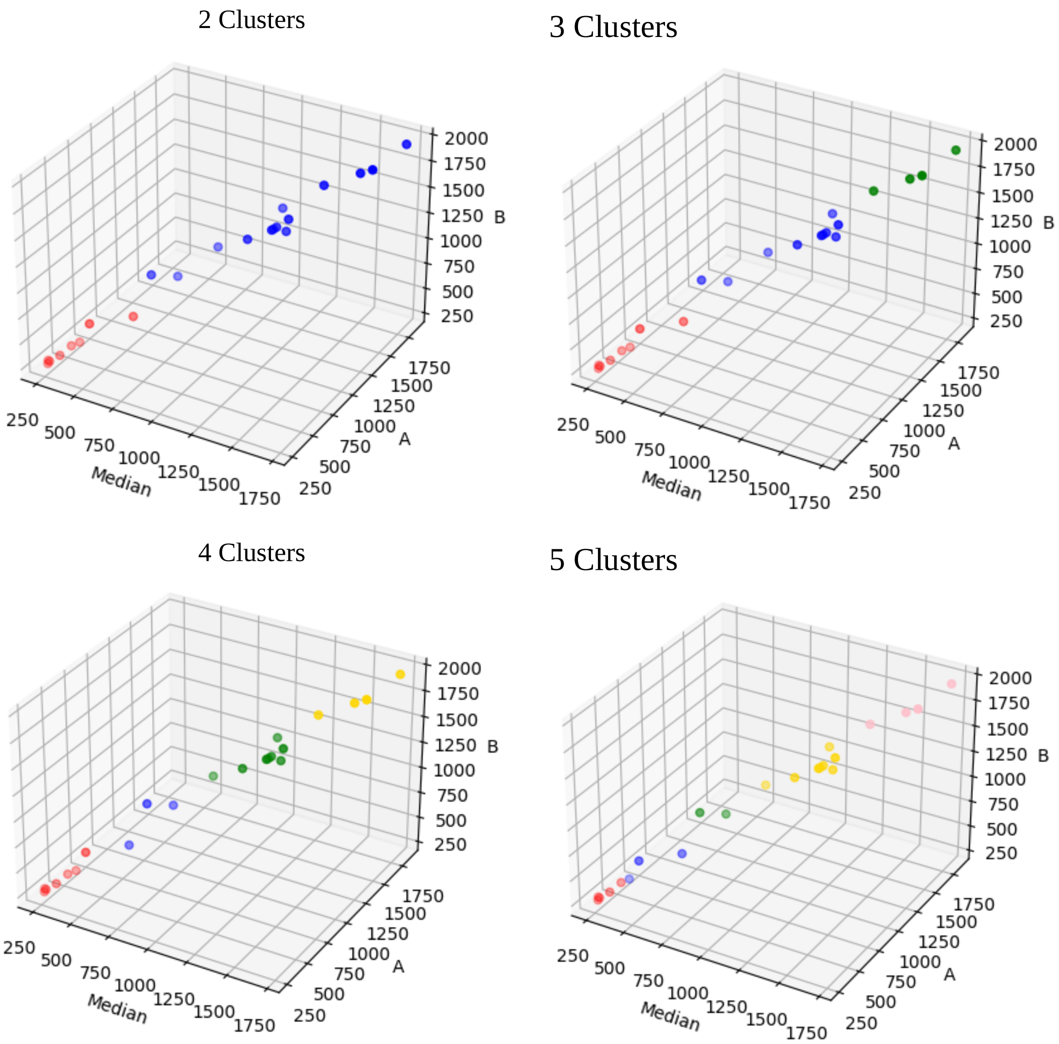


Figure 23 First-phase granules of trend time series by applying Scenario 5

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.7.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	4,785781
Cluster 1	3,77204	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	2,119379	0,135923
Cluster 1	0,008796	0	0,004533
Cluster 2	0,051481	0,031522	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,000096	0,00015	0,000201
Cluster 1	0,000063	0	0,000048	0,000065
Cluster 2	0,000042	0,000018	0	0,000016
Cluster 3	0,000137	0,000059	0,000038	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,000029	0,000045	0,000067	0,000089
Cluster 1	0,000053	0	0,000057	0,000083	0,000111
Cluster 2	0,000151	0,000087	0	0,000087	0,000116
Cluster 3	0,000055	0,000032	0,00002	0	0,000019
Cluster 4	0,000173	0,000099	0,000063	0,000043	0

Table 36 Fuzzy cognitive map weights for Scenario 5 of trend time series

6.1.7.iii Test membership values

Cluster 0	Cluster 1
0,240173	0,759827
0,240173	0,759827
0,240173	0,759827

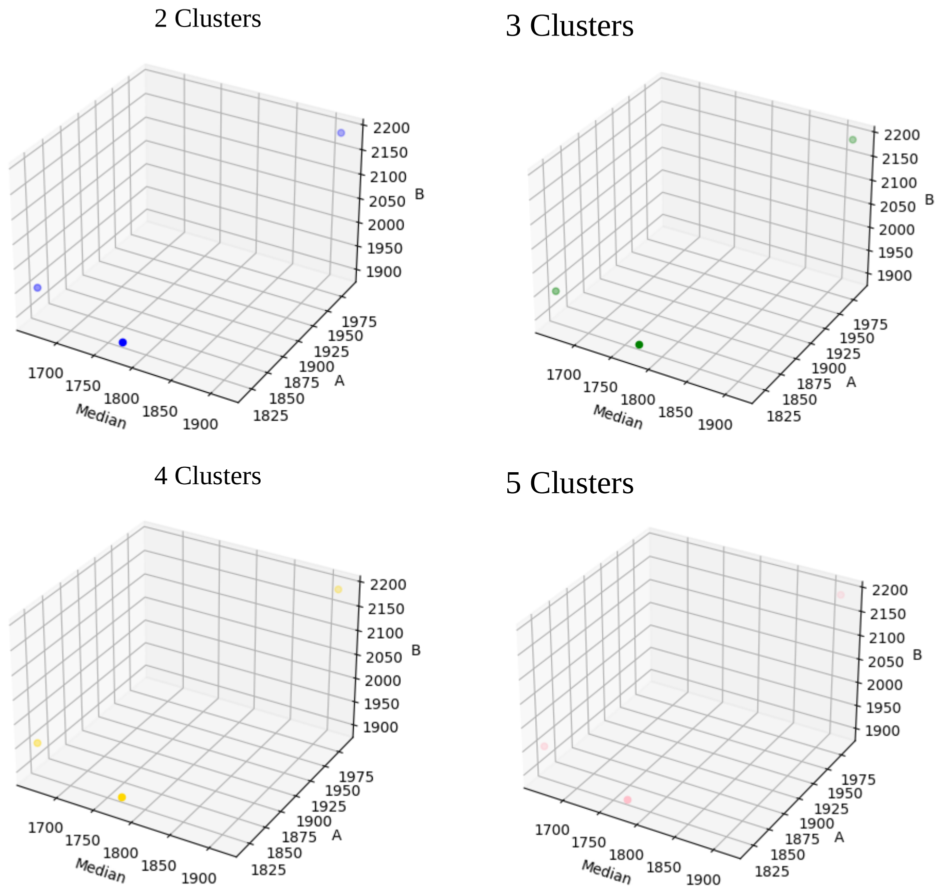
Cluster 0	Cluster 1	Cluster 2
0,120429	0,369418	0,510153
0,120429	0,369418	0,510153
0,120429	0,369418	0,510153

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,084355	0,196725	0,307474	0,411447
0,084355	0,196725	0,307474	0,411447
0,084355	0,196725	0,307474	0,411447

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,067136	0,117030	0,184172	0,270759	0,360903
0,067136	0,117030	0,184172	0,270759	0,360903
0,067136	0,117030	0,184172	0,270759	0,360903

Table 37 Membership degree between first and second-phase granules (clusters) for Scenario 5 of trend time series

6.1.7.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 24 Test first-phase granules values and 3D representation of pr ocess 5 applied to trend time series

6.1.7.v Test membership forecast

Cluster 0	Cluster 1
0,974328	0,712169
0,967963	0,975282
0,990692	0,974697

Cluster 0	Cluster 1	Cluster 2
0,701045	0,500843	0,504461
0,755847	0,502113	0,512967
0,756557	0,502243	0,513681

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,500037	0,500012	0,500003	0,500009
0,500056	0,500022	0,500010	0,500029
0,500056	0,500022	0,500010	0,500029

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,500016	0,500019	0,500021	0,500004	0,500012
0,500029	0,500038	0,500055	0,500016	0,500047
0,500029	0,500038	0,500055	0,500016	0,500047

Table 38 Test membership degree forecast values for Scenario 5 applied to trend time series

6.1.7.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,541255	0,354419	0,309686	0,505731
0,576098	0,421372	0,309714	0,505793
0,609447	0,422313	0,309714	0,505793

Table 39 Error of type 2(membership degree forecast) values for Scenario 5 applied to trend time series

6.1.7.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters

	a	m	b
Test granule 1	769,39	825,38	883,72
Test granule 2	839,94	900,16	961,47
Test granule 3	834,67	894,57	955,65

3 clusters

	a	m	b
Test granule 1	942,58	1006,6	1073,3
Test granule 2	927,74	990,85	1056,8
Test granule 3	927,78	990,9	1056,9

4 clusters

	a	m	b
Test granule 1	948,67	1021,3	1096
Test granule 2	948,67	1021,3	1096
Test granule 3	948,67	1021,3	1096

5 clusters

	a	m	b
Test granule 1	863,79	931,01	1004,5
Test granule 2	863,8	931,02	1004,5
Test granule 3	863,8	931,02	1004,5

Table 40 Test fuzzy triangles with forecasting method 1 for Scenario 5 applied to trend time series

6.1.7.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,548407	0,449272	0,441367	0,489951
0,503125	0,452758	0,436106	0,485145
0,55939	0,511688	0,496853	0,540609

Table 41 Error of type 3 for method 1 for Scenario 5 applied to trend time series

6.1.7.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters

	a	m	b
Test granule 1	362,89	387,92	421,49
Test granule 2	1214,1	1305,1	1390,3
Test granule 3	362,89	387,92	421,49

3 clusters

	a	m	b
Test granule 1	359,22	383,62	416,56
Test granule 2	359,22	383,62	416,56
Test granule 3	359,22	383,62	416,56

4 clusters

	a	m	b
Test granule 1	321,56	346,68	381,47
Test granule 2	321,56	346,68	381,47
Test granule 3	321,56	346,68	381,47

5 clusters

	a	m	b
Test granule 1	725,96	837,64	954,42
Test granule 2	725,96	837,64	954,42
Test granule 3	725,96	837,64	954,42

Table 42 Test fuzzy triangles with forecasting method 2 for Scenario 5 applied to trend time series

6.1.7.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,786401	0,788751	0,808737	0,541204
0,280961	0,786762	0,806936	0,536883
0,807617	0,809734	0,827735	0,586774

Table 43 Error of type 3 for method 2 for Scenario 5 applied to trend time series

6.1.8. Scenario 6– Optimization with IPOPT algorithm

6.1.8.i First-phase granules clusterization

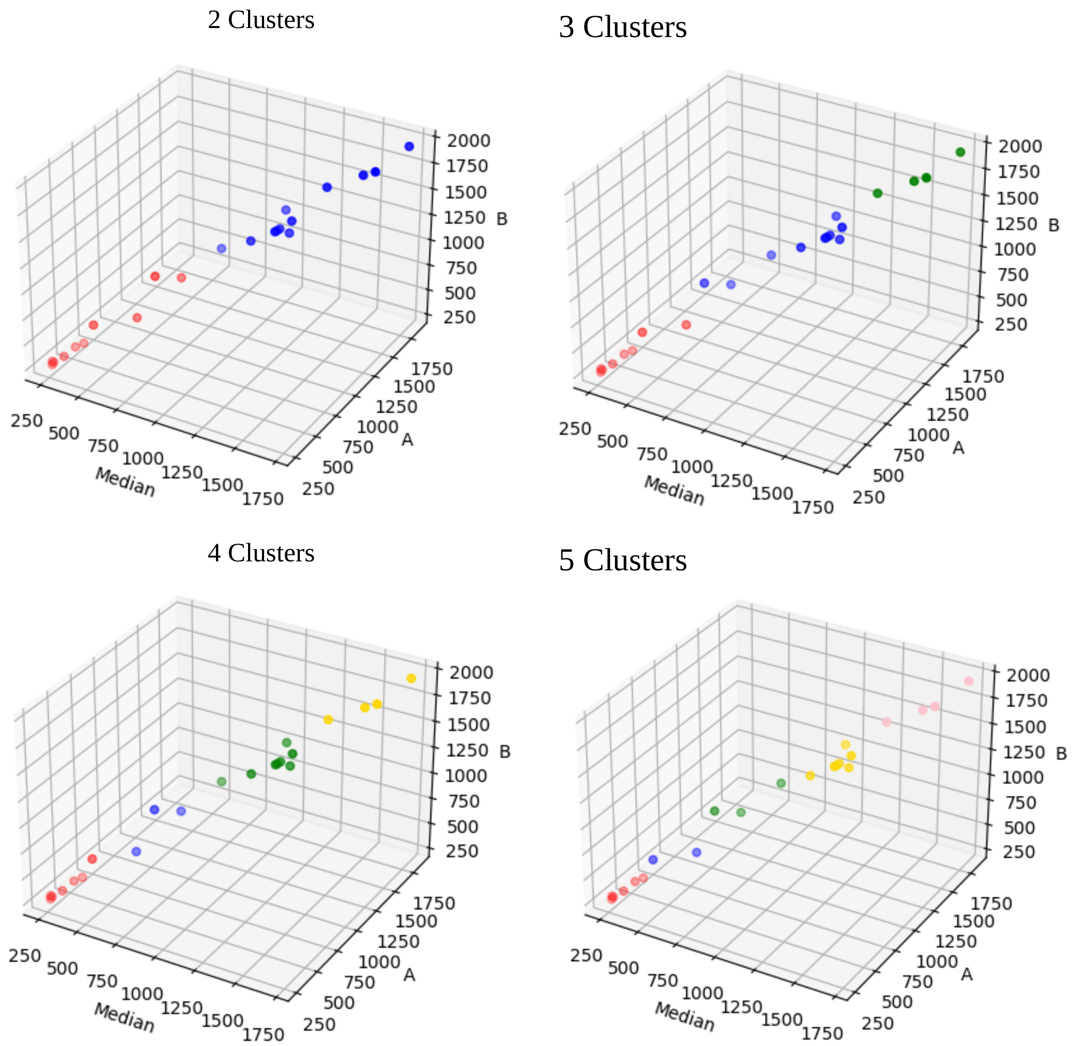


Figure 25 First-phase granules of trend time series by applying Scenario 6

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.1.8.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	15,363953
Cluster 1	18,73015	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	0,000014	0,000022
Cluster 1	0,000016	0	0,000011
Cluster 2	0,000085	0,000032	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,000019	0,000023	0,00003
Cluster 1	0,000047	0	0,000049	0,000069
Cluster 2	0,000026	0,000018	0	0,000018
Cluster 3	0,000073	0,000053	0,000042	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,000026	0,000029	0,000035	0,000043
Cluster 1	0,000082	0	0,000083	0,000102	0,000134
Cluster 2	0,000067	0,000058	0	0,000058	0,000076
Cluster 3	0,000033	0,000029	0,000025	0	0,000025
Cluster 4	0,000077	0,000069	0,000061	0,000054	0

Table 44 Fuzzy cognitive map weights for Scenario 6 of trend time series

6.1.8.iii Test membership values

Cluster 0	Cluster 1
0,249266	0,750734
0,245215	0,754785
0,294176	0,705824

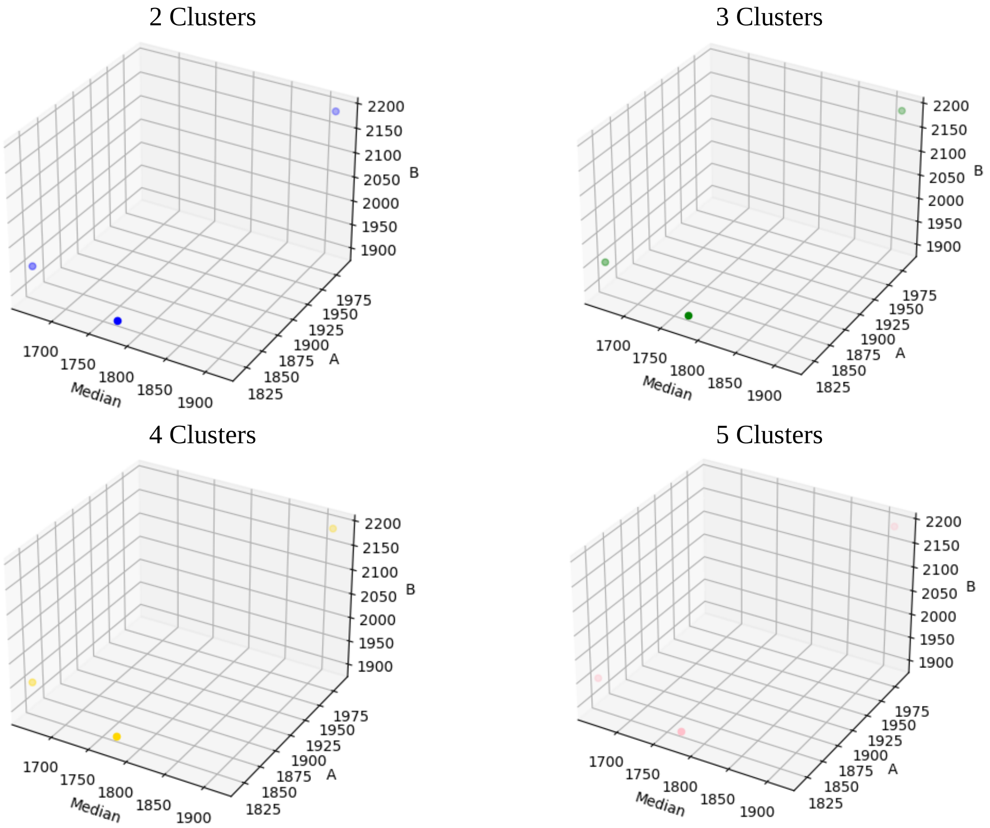
Cluster 0	Cluster 1	Cluster 2
0,177766	0,360023	0,462211
0,174311	0,360405	0,465284
0,210848	0,354084	0,435068

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,180320	0,227632	0,274324	0,317725
0,178876	0,227377	0,274630	0,319117
0,194157	0,232165	0,269310	0,304369

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,165503	0,178493	0,195894	0,218447	0,241664
0,164842	0,178102	0,195962	0,218671	0,242422
0,171790	0,182422	0,196711	0,214971	0,234106

Table 45 Membership degree between first and second-phase granules (clusters) for Scenario 6 of trend time series

6.1.8.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	1775,14	1818,89	1894,29
Test granule 2	1664,45	1817,06	1955,61
Test granule 3	1915,95	1988,12	2189,49

Figure 26 Test first-phase granules values and 3D representation of pr ocess 6 applied to trend time series

6.1.8.v Test membership forecast

Cluster 0	Cluster 1
0,9999895295	0,9914296963
0,9999997575	0,9999999927
0,9999997874	0,9999999927

Cluster 0	Cluster 1	Cluster 2
0,5000037999	0,5000020244	0,5000066602
0,5000045257	0,5000034310	0,5000145354
0,5000045258	0,5000034311	0,5000145354

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,5000050367	0,5000109530	0,5000036448	0,5000092461
0,5000089979	0,5000206274	0,5000077901	0,5000210854
0,5000089981	0,5000206277	0,5000077902	0,5000210857

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,5000071071	0,5000210890	0,5000130889	0,5000053896	0,5000122053
0,5000166841	0,5000500674	0,5000323096	0,5000139837	0,5000326040
0,5000166847	0,5000500689	0,5000323106	0,5000139842	0,5000326051

Table 46 Test membership degree forecast values for Scenario 6 applied to trend time series

6.1.8.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,621521	0,124860	0,260548	0,453781
0,629831	0,126770	0,260982	0,453968
0,584727	0,109122	0,256794	0,452591

Table 47 Error of type 2(membership degree forecast) values for Scenario 6 applied to trend time series

6.1.8.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	836,37	896,37	957,53	Test granule 1	1018,4	1087,2	1157,8
Test granule 2	838,27	898,39	959,63	Test granule 2	1018,4	1087,2	1157,8
Test granule 3	838,27	898,39	959,63	Test granule 3	1018,4	1087,2	1157,8

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	948,68	1021,3	1096	Test granule 1	863,79	931,02	1004,5
Test granule 2	948,68	1021,3	1096	Test granule 2	863,79	931,02	1004,5
Test granule 3	948,68	1021,3	1096	Test granule 3	863,79	931,02	1004,5

Table 48 Test fuzzy triangles with forecasting method 1 for Scenario 6 applied to trend time series

6.1.8.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,50982	0,405381	0,441361	0,48995
0,504096	0,39978	0,436099	0,485147
0,557518	0,46444	0,496847	0,540611

Table 49 Error of type 3 for method 1 for Scenario 6 applied to trend time series

6.1.8.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	426,93	467,62	516,99	Test granule 1	1559,2	1660,1	1770,7
Test granule 2	1290,8	1378,8	1458,6	Test granule 2	1559,2	1660,1	1770,7
Test granule 3	1290,8	1378,8	1458,6	Test granule 3	1559,2	1660,1	1770,7

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	688,69	771,77	859,86	Test granule 1	512,9	546,69	614,76
Test granule 2	1562,9	1663,6	1773,8	Test granule 2	512,9	546,69	614,76
Test granule 3	1562,9	1663,6	1773,8	Test granule 3	512,9	546,69	614,76

Table 50 Test fuzzy triangles with forecasting method 2 for Scenario 6 applied to trend time series

6.1.8.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,74281	0,090796	0,577226	0,694925
0,240738	0,082235	0,08034	0,692052
0,322531	0,181103	0,179412	0,725226

Table 51 Error of type 3 for method 2 for Scenario 6 applied to trend time series

6.1.9. Summary tables

6.1.9.i Cluster forecast tables

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	Cluster 1	Cluster 0	Cluster 1
Test granule 2	Cluster 1	Cluster 1	Cluster 1
Test granule 3	Cluster 1	Cluster 1	Cluster 1
3 CLUSTERS			
Test granule 1	Cluster 2	Cluster 1	Cluster 0
Test granule 2	Cluster 2	Cluster 2	Cluster 0
Test granule 3	Cluster 2	Cluster 2	Cluster 0
4 CLUSTERS			
Test granule 1	Cluster 3	Cluster 2	**Cluster 3**
Test granule 2	Cluster 3	Cluster 2	**Cluster 3**
Test granule 3	Cluster 3	Cluster 2	**Cluster 3**
5 CLUSTERS			
Test granule 1	Cluster 4	Cluster 3	Cluster 4
Test granule 2	Cluster 3	Cluster 3	Cluster 3
Test granule 3	Cluster 3	Cluster 3	Cluster 3

Table 52 Second-phase granules forecast accuracy for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	Cluster 1	Cluster 0	Cluster 0
Test granule 2	Cluster 1	Cluster 1	Cluster 1
Test granule 3	Cluster 1	Cluster 0	Cluster 1
3 CLUSTERS			
Test granule 1	Cluster 2	Cluster 0	Cluster 2
Test granule 2	Cluster 2	Cluster 0	Cluster 2
Test granule 3	Cluster 2	Cluster 0	Cluster 2
4 CLUSTERS			
Test granule 1	Cluster 0	Cluster 0	Cluster 1
Test granule 2	Cluster 1	Cluster 0	Cluster 3
Test granule 3	Cluster 1	Cluster 0	Cluster 3
5 CLUSTERS			
Test granule 1	Cluster 1	Cluster 2	Cluster 1
Test granule 2	Cluster 3	Cluster 2	Cluster 1
Test granule 3	Cluster 3	Cluster 2	Cluster 1

Table 53 Second-phase granules forecast accuracy for Scenarios optimized by IPOPT algorithm of the trend time series

6.2.1.i Error type 2 tables

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	0,065997	0,470557	0,356979
Test granule 2	0,080320	0,545939	0,438391
Test granule 3	0,080393	0,668784	0,410233
	3 CLUSTERS		
Test granule 1	0,037497	0,634404	0,107196
Test granule 2	0,031341	0,657661	0,092908
Test granule 3	0,036219	0,772574	0,074141
	4 CLUSTERS		
Test granule 1	0,014354	0,856382	**0,011165**
Test granule 2	0,012928	1,025407	**0,006167**
Test granule 3	0,016770	1,193808	**0,007953**
	5 CLUSTERS		
Test granule 1	0,025723	1,101291	0,004173
Test granule 2	0,017681	1,469028	0,001369
Test granule 3	0,021003	1,696927	0,004283

Table 54 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
	2 CLUSTERS		
Test granule 1	0,124977	0,541255	0,621521
Test granule 2	0,125041	0,576098	0,629831
Test granule 3	0,125041	0,609447	0,584727
	3 CLUSTERS		
Test granule 1	0,159826	0,354419	0,124860
Test granule 2	0,159827	0,421372	0,126770
Test granule 3	0,159827	0,422313	0,109122
	4 CLUSTERS		
Test granule 1	0,306587	0,309686	0,260548
Test granule 2	0,306600	0,309714	0,260982
Test granule 3	0,306600	0,309714	0,256794
	5 CLUSTERS		
Test granule 1	0,502717	0,505731	0,453781
Test granule 2	0,502771	0,505793	0,453968
Test granule 3	0,502771	0,505793	0,452591

Table 55 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the trend time series

6.2.1.ii Error type 3 tables (Method 1)

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,462086759	0,54851	0,481712072
Test granule 2	0,433597199	0,503135822	0,484736405
Test granule 3	0,494646807	0,559422848	0,540624856
3 CLUSTERS			
Test granule 1	0,355447569	0,461642523	0,410274911
Test granule 2	0,349344874	0,48446466	0,413616032
Test granule 3	0,423622644	0,533210102	0,476801738
4 CLUSTERS			
Test granule 1	0,374223806	0,404145991	0,412463195
Test granule 2	0,36755856	0,41258381	0,41485382
Test granule 3	0,439399957	0,473504033	0,481636351
5 CLUSTERS			
Test granule 1	0,432022914	0,499670408	0,46953166
Test granule 2	0,413264743	0,511131022	0,466206374
Test granule 3	0,412422673	0,553767656	0,527003919

Table 56 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,423639657	0,548406434	0,509819034
Test granule 2	0,418188306	0,503124816	0,504095918
Test granule 3	0,480865044	0,559388929	0,557518101
3 CLUSTERS			
Test granule 1	0,405391814	0,449288671	0,405391814
Test granule 2	0,399792537	0,452763595	0,399792537
Test granule 3	0,464450994	0,511684467	0,464450994
4 CLUSTERS			
Test granule 1	0,441362749	0,441364571	0,441362749
Test granule 2	0,436102201	0,43610404	0,436102201
Test granule 3	0,496849133	0,496850774	0,496849133
5 CLUSTERS			
Test granule 1	0,48995321	0,48995321	0,489951388
Test granule 2	0,485150227	0,485146548	0,485148387
Test granule 3	0,540613369	0,540610087	0,540611728

Table 57 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the trend time series

6.2.1.i Error type 3 tables (Method 2)

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,335195563	0,786401453	0,247820827
Test granule 2	0,328935266	0,280964904	0,240737744
Test granule 3	0,401226953	0,807617029	0,322530672
3 CLUSTERS			
Test granule 1	0,20556777	0,788752014	0,788751385
Test granule 2	0,1980868	0,786762745	0,786762109
Test granule 3	0,28447438	0,809734122	0,809733555
4 CLUSTERS			
Test granule 1	0,180036751	0,088920651	**0,088919742**
Test granule 2	0,172315362	0,080341245	**0,080340327**
Test granule 3	0,261479217	0,179413182	**0,179412363**
5 CLUSTERS			
Test granule 1	0,179926357	0,830948044	0,088719317
Test granule 2	0,341482679	0,829356124	0,304392767
Test granule 3	0,412422673	0,84773905	0,37932834

Table 58 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by genetic algorithm of the trend time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,335188181	0,786400939	0,742810186
Test granule 2	0,328927815	0,280961244	0,240737744
Test granule 3	0,401220305	0,807616566	0,322530672
3 CLUSTERS			
Test granule 1	0,20556746	0,788751385	0,090796455
Test granule 2	0,198086487	0,786762109	0,082234712
Test granule 3	0,284474101	0,809733555	0,181102672
4 CLUSTERS			
Test granule 1	0,835561702	0,808737464	0,577225818
Test granule 2	0,698732049	0,806936393	0,080340327
Test granule 3	0,731186695	0,827734526	0,179412363
5 CLUSTERS			
Test granule 1	0,843425675	0,541203866	0,694924859
Test granule 2	0,341489612	0,536883497	0,692052042
Test granule 3	0,412428859	0,586773577	0,725226304

Table 59 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the trend time series

6.3. Time series differentiated

6.3.1. Errors of Fuzzy C-means

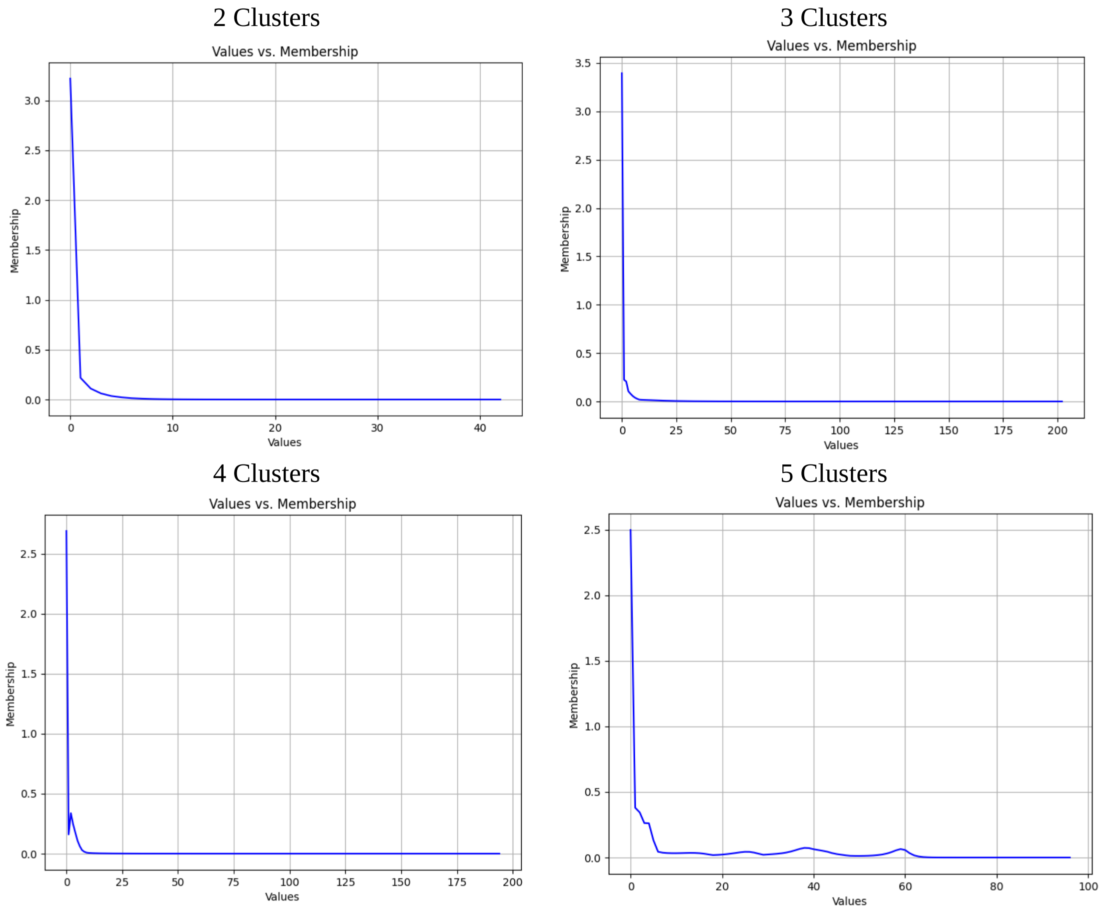


Figure 27 Stationary time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

6.3.2. First-phase granules (blue) and second-phase granules (red)

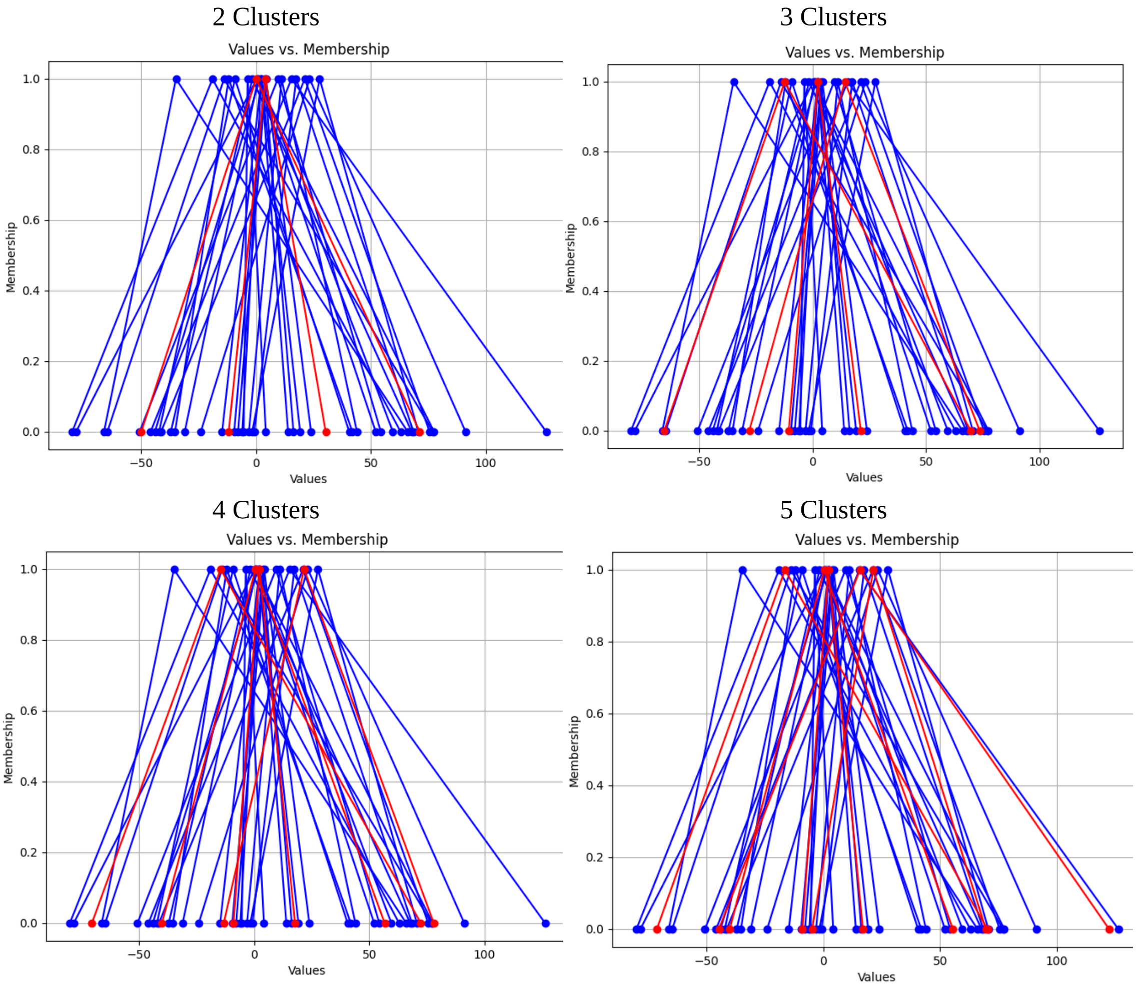


Figure 28 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the stationary time series

6.3.3. Scenario 1 – Optimization with genetic algorithm

6.3.3.i First-phase granules clusterization

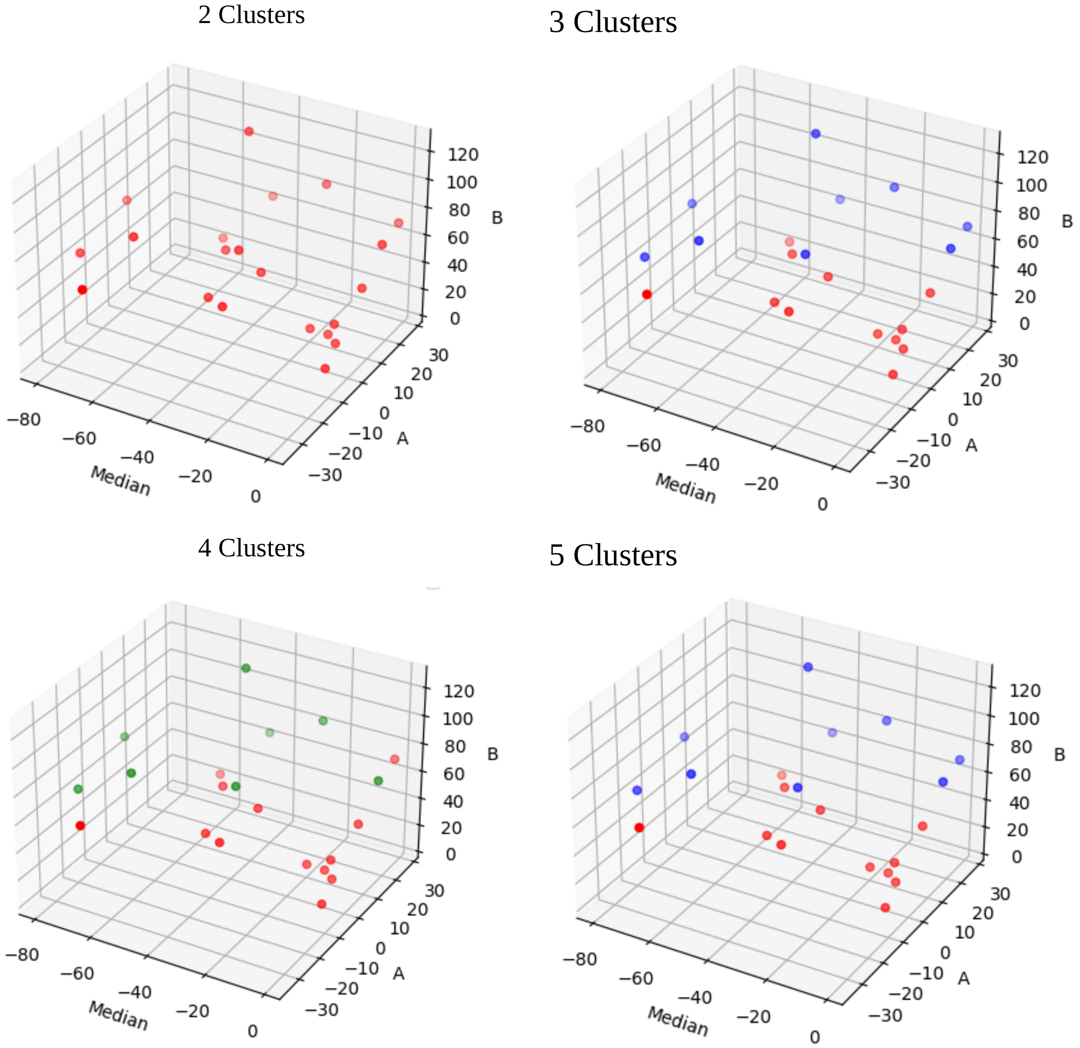


Figure 29 First-phase granules of stationary time series by applying Scenario 1

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.3.3.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	2,101279
Cluster 1	-2,581747	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	-0,096047	-1,643742
Cluster 1	-0,281185	0	-0,730151
Cluster 2	0,860308	1,417328	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,851491	-2,72959	-2,058962
Cluster 1	-1,781147	0	-0,994817	1,27063
Cluster 2	-2,135727	0,067329	0	0,473758
Cluster 3	1,564022	-1,022913	2,635414	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-1,88407	-2,063965	-2,133642	-0,479826
Cluster 1	-0,635924	0	-1,7395	-1,72305	-1,50647
Cluster 2	0,773105	5,042966	0	0,808169	1,226744
Cluster 3	-0,621714	-1,42084	0,967256	0	0,367148
Cluster 4	-0,135308	-0,112375	0,680209	-1,312128	0

Table 60 Fuzzy cognitive map weights for Scenario 1 of stationary time series

6.3.3.iii Test membership values

Cluster 0	Cluster 1
0,701598	0,298402
0,701598	0,298402
0,701598	0,298402

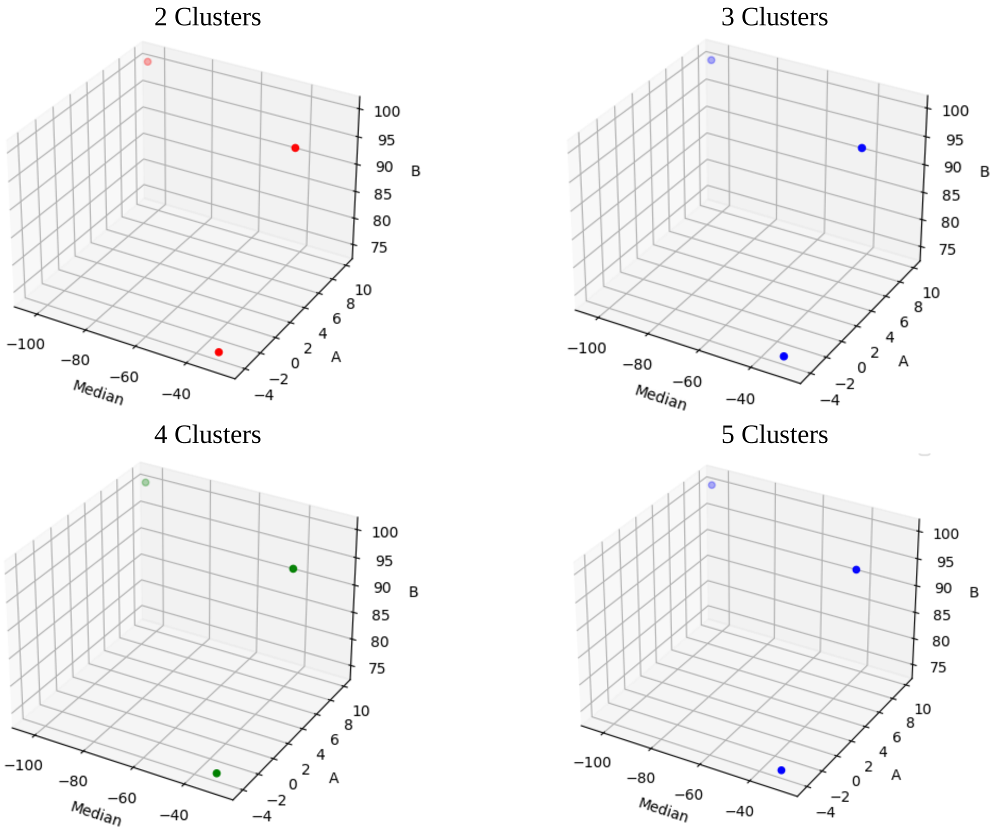
Cluster 0	Cluster 1	Cluster 2
0,423546	0,447386	0,129067
0,423546	0,447386	0,129067
0,423546	0,447386	0,129067

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,327553	0,256365	0,335843	0,080239
0,322084	0,252085	0,346932	0,078900
0,322084	0,252085	0,346932	0,078900

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,245161	0,259107	0,193715	0,059114	0,242903
0,224784	0,320688	0,177613	0,054201	0,222714
0,224704	0,320930	0,177550	0,054181	0,222634

Table 61 Membership degree between first and second-phase granules (clusters) for Scenario 1 of stationary time series

6.3.3.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 30 Test first-phase granules values and 3D representation of pr ocess 1 applied to stationary time series

6.3.3.v Test membership forecast

Cluster 0	Cluster 1
0,6518145654	0,1404751265
0,5732631724	0,1567224112
0,5815933001	0,1854243621

Cluster 0	Cluster 1	Cluster 2
0,4365628638	0,4468680145	0,7307582724
0,2237234228	0,3415670621	0,7328120188
0,2248957943	0,3548080312	0,6629757249

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,2901236232	0,3060694365	0,3467102196	0,7613678797
0,0950575292	0,5264131278	0,4407019619	0,7416193628
0,0926528114	0,5828769220	0,5458145343	0,6838732114

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,2246655382	0,3051411424	0,9089476392	0,3982587792	0,4942405738
0,0282546826	0,0409040115	0,9334338368	0,6194962322	0,5077541385
0,0274099220	0,0300606847	0,7944014637	0,7337253293	0,4535474525

Table 62 Test membership degree forecast values for Scenario 1 applied to stationary time series

6.3.3.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,027419	0,362202	0,467926	0,692287
0,036543	0,415635	0,574787	1,088974
0,027165	0,333091	0,567610	1,019136

Table 63 Error of type 2(membership degree forecast) values for Scenario 1 applied to stationary time series

6.3.3.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-43,166	0,7914	63,974	Test granule 1	-30,133	1,6808	48,834
Test granule 2	-41,745	0,9423	62,444	Test granule 2	-24,518	2,9476	43,363
Test granule 3	-40,716	1,0516	61,336	Test granule 3	-25,38	3,0879	44,954

	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-25,867	3,0593	46,268	Test granule 1	-30,881	5,5284	62,042
Test granule 2	-22,302	5,6378	46,676	Test granule 2	-23,114	5,9769	49,098
Test granule 3	-22,667	6,4973	49,541	Test granule 3	-21,518	5,6914	45,919

Table 64 Test fuzzy triangles with forecasting method 1 for Scenario 1 applied to stationary time series

6.3.3.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,253647	0,291786	0,367025	0,202188
0,508579	0,668919	0,651218	0,634513
0,424285	0,440859	0,437434	0,46763

Table 65 Error of type 3 for method 1 for Scenario 1 applied to stationary time series

6.3.3.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-33,59	2,1537	55,65	Test granule 1	-10,505	2,3332	21,234
Test granule 2	-33,59	2,1537	55,65	Test granule 2	-10,505	2,3332	21,234
Test granule 3	-33,59	2,1537	55,65	Test granule 3	-10,505	2,3332	21,234

	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-8,9864	2,3064	17,727	Test granule 1	-40,069	0,3681	55,472
Test granule 2	-8,9864	2,3064	17,727	Test granule 2	-40,069	0,3681	55,472
Test granule 3	-8,9864	2,3064	17,727	Test granule 3	-40,069	0,3681	55,472

Table 66 Test fuzzy triangles with forecasting method 2 for Scenario 1 applied to stationary time series

6.3.3.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,254663	0,730573	0,776368	0,299298
0,57279	0,840735	0,864348	0,55168
0,40497	0,740525	0,778997	0,469264

Table 67 Error of type 3 for method 2 for Scenario 1 applied to stationary time series

6.3.4. Scenario 2– Optimization with genetic algorithm

6.3.4.i First-phase granules clusterization

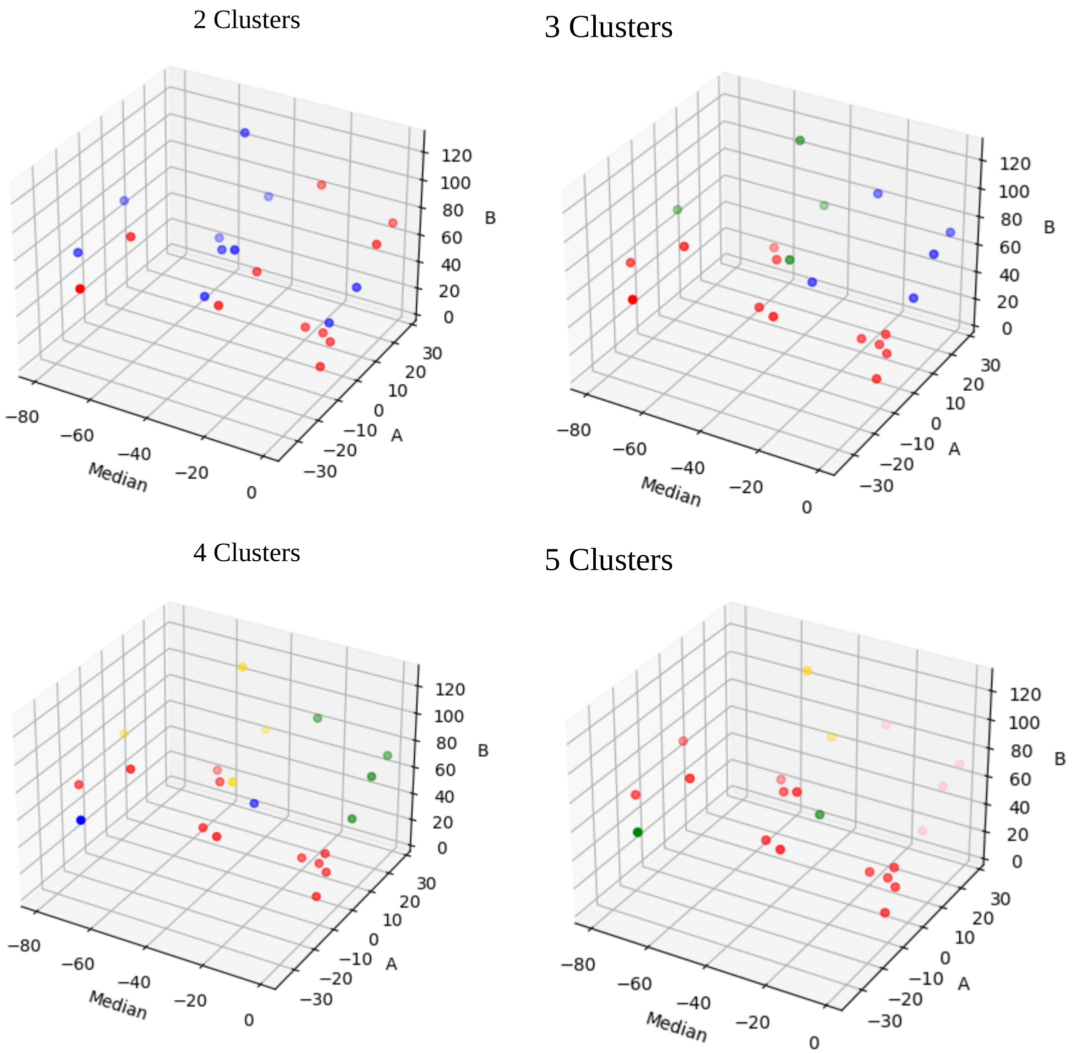


Figure 31 First-phase granules of stationary time series by applying Scenario 2

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.3.4.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	-0,437435
Cluster 1	-0,014413	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	12,818054	-2,352831
Cluster 1	-0,142049	0	0,074045
Cluster 2	0,134228	-0,51075	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	9,257659	2,025973	2,710412
Cluster 1	-0,192064	0	0,408875	0,819957
Cluster 2	-0,489749	0,601575	0	2,186029
Cluster 3	1,539664	-1,568241	-6,189006	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	4,418475	5,058963	1,724995	1,823097
Cluster 1	-0,423534	0	1,019727	1,508819	0,933831
Cluster 2	-0,549684	1,038608	0	2,339394	0,406731
Cluster 3	0,309128	-0,978948	-1,65418	0	-0,237767
Cluster 4	-2,317337	-1,02294	1,511328	3,906917	0

Table 68 Fuzzy cognitive map weights for Scenario 2 of stationary time series

6.3.4.iii Test membership values

Cluster 0	Cluster 1
1,215976	-0,215976
0,433230	0,566770
0,775301	0,224699

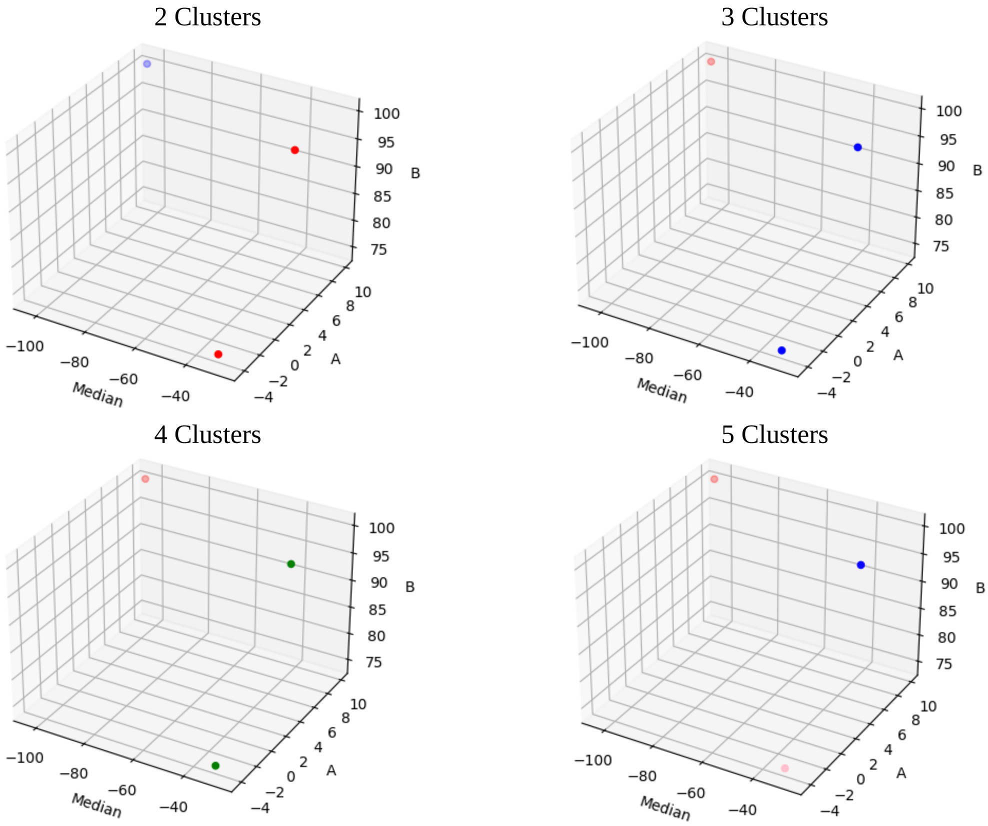
Cluster 0	Cluster 1	Cluster 2
-0,868312	2,767028	-0,898717
0,498879	0,112939	0,388182
-0,235785	1,319685	-0,083900

Cluster 0	Cluster 1	Cluster 2	Cluster 3
-1,988968	1,749745	2,820197	-1,580975
0,414039	0,237508	0,052420	0,296033
-0,329429	0,450707	1,025890	-0,147168

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
-2,735882	1,456605	1,575237	-1,649721	2,353761
0,435900	-0,022771	0,232241	0,284032	0,070598
-0,269966	0,645236	0,244689	-0,099343	0,479384

Table 69 Membership degree between first and second-phase granules (clusters) for Scenario 2 of stationary time series

6.3.4.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 32 Test first-phase granules values and 3D representation of pr ocess 2 applied to stationary time series

6.3.4.v Test membership forecast

Cluster 0	Cluster 1
0,4273757156	0,4988066514
0,4456665506	0,4984600167
0,4457040108	0,4983941089

Cluster 0	Cluster 1	Cluster 2
0,5899655995	0,4930948676	0,5001907204
0,9941975285	0,4883102778	0,4569424287
0,9944261442	0,4731781863	0,4710437327

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,9614804025	0,5616895527	0,6795439426	0,4671741633
0,9996076261	0,6168610871	0,7085331281	0,0264367255
0,9992671332	0,5298122092	0,4848353083	0,0215954197

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,7744147786	0,6862603083	0,6241663100	0,4740848109	0,7634926210
0,9997750873	0,8502762702	0,8464050288	0,1615910418	0,5741851364
0,9999142848	0,7720049330	0,7201301007	0,1130511392	0,2182184495

Table 70 Test membership degree forecast values for Scenario 2 applied to stationary time series

6.3.4.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,132805023	9,25428826	18,893930	20,859655
0,004820979	0,39097204	0,989967	1,725956
0,183543465	2,53795579	2,092913	1,968028

Table 71 Error of type 2(membership degree forecast) values for Scenario 2 applied to stationary time series

6.3.4.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-29,431	2,25	49,186	Test granule 1	-36,334	0,603	55,596
Test granule 2	-29,834	2,2072	49,62	Test granule 2	-42,97	-2,1459	59,254
Test granule 3	-29,836	2,207	49,622	Test granule 3	-42,855	-2,2422	58,866
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-38,698	0,7921	60,918	Test granule 1	-35,735	4,7225	70,448
Test granule 2	-44,497	0,5115	69,308	Test granule 2	-42,915	2,8927	76,863
Test granule 3	-48,21	-1,8152	69,022	Test granule 3	-48,385	0,3255	78,511

Table 72 Test fuzzy triangles with forecasting method 1 for Scenario 2 applied to stationary time series

6.3.4.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,300209075	0,266082996	0,240721785	0,162246103
0,618283232	0,532197461	0,465638427	0,426589681
0,421841204	0,484464941	0,444649346	0,357427988

Table 73 Error of type 3 for method 1 for Scenario 2 applied to stationary time series

6.3.4.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-40,182	3,4553	60,222	Test granule 1	-37,797	-6,2161	42,27
Test granule 2	-40,182	3,4553	60,222	Test granule 2	-37,797	-6,2161	42,27
Test granule 3	-40,182	3,4553	60,222	Test granule 3	-37,797	-6,2161	42,27
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-35,86	-4,025	40,515	Test granule 1	-39,03	-3,67	44,94
Test granule 2	-35,86	-4,025	40,515	Test granule 2	-39,03	-3,67	44,94
Test granule 3	-35,86	-4,025	40,515	Test granule 3	-39,03	-3,67	44,94

Table 74 Test fuzzy triangles with forecasting method 2 for Scenario 2 applied to stationary time series

6.3.4.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,285096	0,375697	0,357527	0,349275
0,514519	0,654788	0,661803	0,624609
0,410439	0,602647	0,584575	0,572314

Table 75 Error of type 3 for method 2 for Scenario 2 applied to stationary time series

6.3.5. Scenario 3– Optimization with genetic algorithm

6.3.5.i First-phase granules clusterization

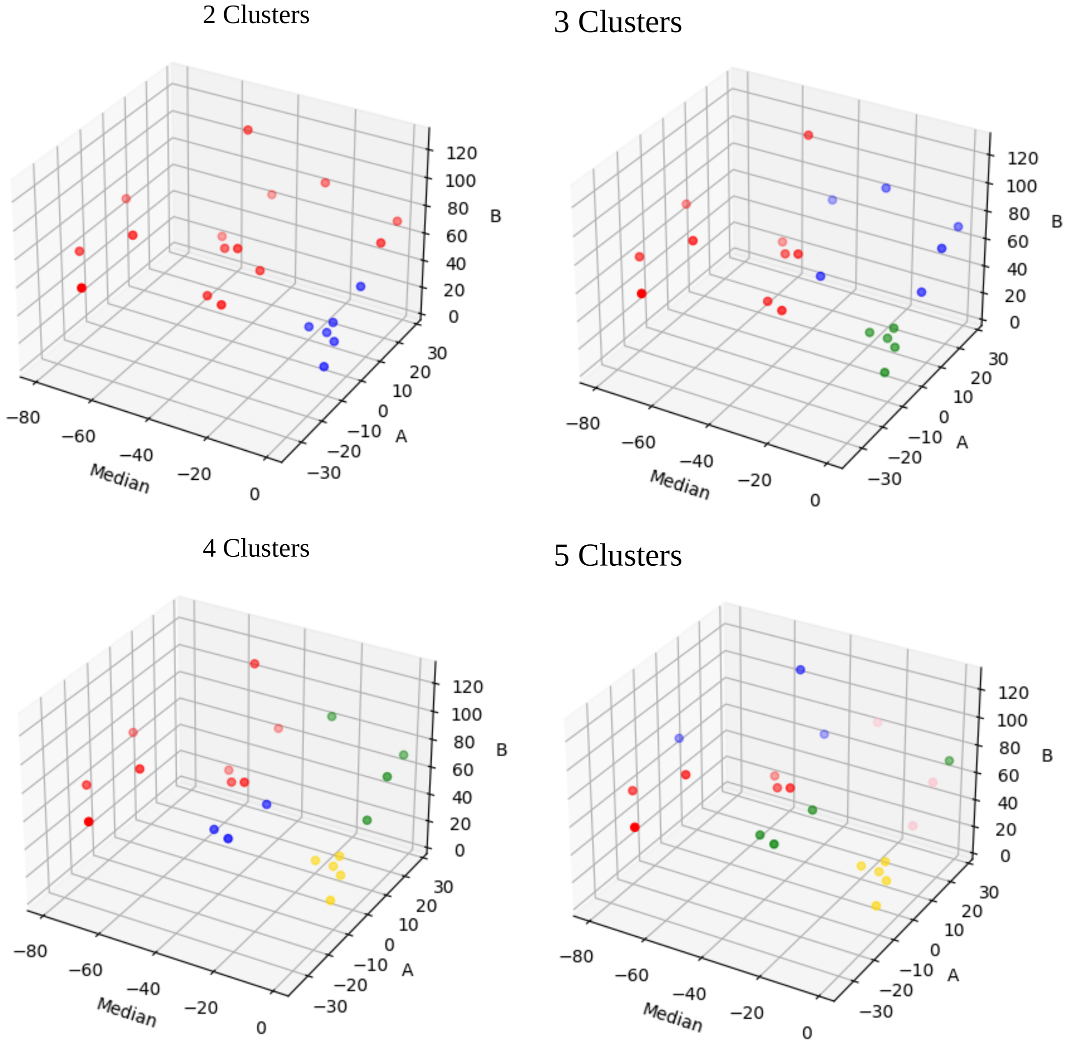


Figure 33 First-phase granules of stationary time series by applying Scenario 3

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.3.5.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	1,628044
Cluster 1	6,317825	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	-0,585302	-0,559657
Cluster 1	-1,662593	0	0,241912
Cluster 2	0,024684	-0,480841	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	-0,514574	-1,093083	-2,236602
Cluster 1	-0,861701	0	-1,389953	-1,499022
Cluster 2	-3,277089	-0,015851	0	-0,959109
Cluster 3	-1,024532	-0,938256	-1,457409	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-2,11666	-1,64501	-1,418611	-1,161918
Cluster 1	-1,239649	0	-2,247942	-1,879085	-0,655807
Cluster 2	-1,795475	-0,721092	0	-1,640153	-2,240778
Cluster 3	-1,238307	-0,91237	-3,13439	0	-0,550261
Cluster 4	-1,781039	-1,941555	-0,867738	-1,986647	0

Table 76 Fuzzy cognitive map weights for Scenario 3 of stationary time series

6.3.5.iii Test membership values

Cluster 0	Cluster 1
0,709147	0,290853
0,651008	0,348992
0,657878	0,342122

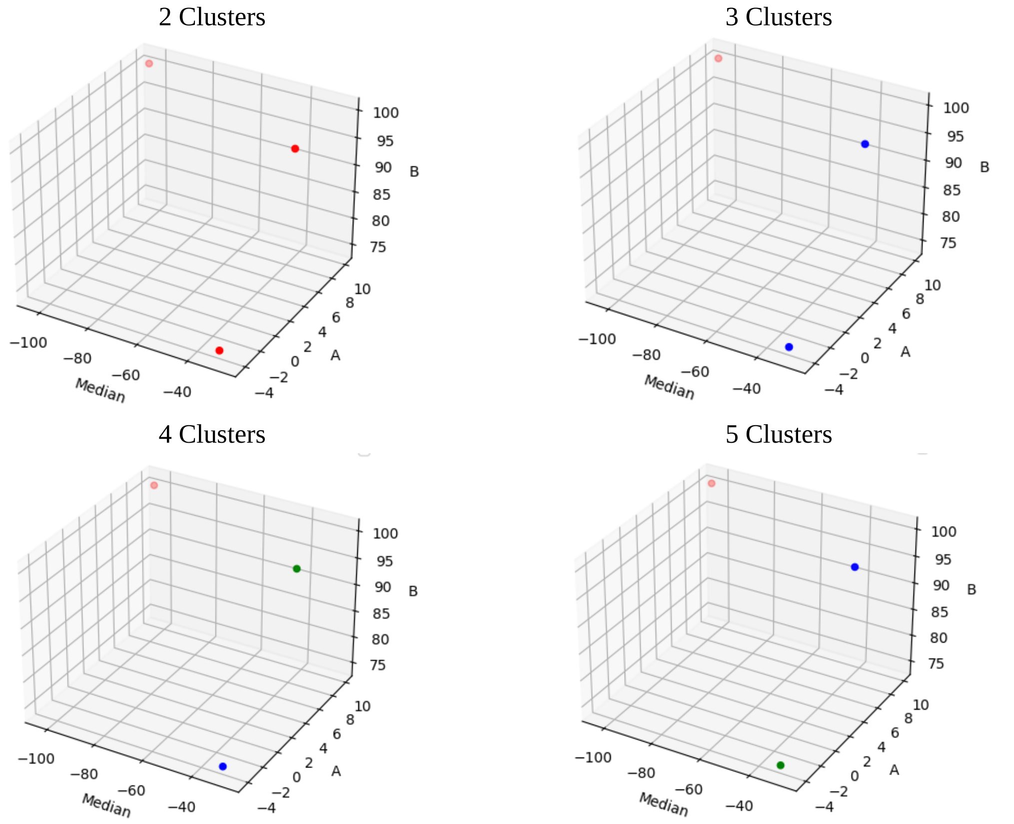
Cluster 0	Cluster 1	Cluster 2
0,339496	0,416353	0,244152
0,395122	0,343181	0,261697
0,339438	0,411357	0,249205

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,244364	0,289048	0,265156	0,201431
0,285487	0,258719	0,243165	0,212630
0,248872	0,263054	0,284767	0,203307

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,200873	0,187651	0,225218	0,178767	0,207492
0,219631	0,212609	0,203760	0,175471	0,188529
0,195262	0,220789	0,205257	0,168502	0,210191

Table 77 Membership degree between first and second-phase granules (clusters) for Scenario 3 of stationary time series

6.3.5.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 34 Test first-phase granules values and 3D representation of pr ocess 3 applied to stationary time series

6.3.5.v Test membership forecast

Cluster 0	Cluster 1
0,6384876967	0,9838650764
0,8322664670	0,9826020798
0,8319792253	0,9948218403

Cluster 0	Cluster 1	Cluster 2
0,4131910178	0,3579512494	0,4603419256
0,3852926532	0,3599441950	0,4596086517
0,3851135940	0,3706568119	0,4591996311

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,2936837630	0,2877469554	0,2439293579	0,2903488317
0,2565273055	0,2635968089	0,2234861318	0,2836610065
0,2661263378	0,2774994911	0,2465862049	0,3024090177

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,2203130794	0,2375253096	0,2219587482	0,2307938798	0,2090505111
0,1918256386	0,2070435190	0,1956326490	0,2141206178	0,1817311439
0,2184078323	0,2316355985	0,2223324393	0,2423886825	0,2077115426

Table 78 Test membership degree forecast values for Scenario 3 applied to stationary time series

6.3.5.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,485258	0,055580	0,010791	0,005585
0,434316	0,039547	0,006295	0,002410
0,456329	0,047841	0,011785	0,006410

Table 79 Error of type 2(membership degree forecast) values for Scenario 3 applied to stationary time series

6.3.5.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-26,85	2,5241	46,407	Test granule 1	-33,939	0,8969	52,71
Test granule 2	-29,323	2,2616	49,069	Test granule 2	-33,217	1,2222	52,371
Test granule 3	-29,203	2,2743	48,94	Test granule 3	-33,173	1,3384	52,566
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-34,07	1,65	55,311	Test granule 1	-34,176	4,6221	67,235
Test granule 2	-33,196	1,8592	54,457	Test granule 2	-33,866	4,5766	66,55
Test granule 3	-32,771	2,1246	54,509	Test granule 3	-33,834	4,5525	66,414

Table 80 Test fuzzy triangles with forecasting method 1 for Scenario 3 applied to stationary time series

6.3.5.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,351834	0,273279	0,257549	0,17649
0,622996	0,594214	0,581584	0,509227
0,421699	0,431443	0,407601	0,313742

Table 81 Error of type 3 for method 1 for Scenario 3 applied to stationary time series

6.3.5.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-8,335	3,4597	21,691	Test granule 1	-8,9778	1,8864	16,478
Test granule 2	-8,335	3,4597	21,691	Test granule 2	-8,9778	1,8864	16,478
Test granule 3	-8,335	3,4597	21,691	Test granule 3	-8,9778	1,8864	16,478
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-57,822	-3,568	73,905	Test granule 1	-52,53	13,171	100,4
Test granule 2	-8,7247	1,8819	15,893	Test granule 2	-8,721	1,8547	15,76
Test granule 3	-8,7247	1,8819	15,893	Test granule 3	-8,721	1,8547	15,76

Table 82 Test fuzzy triangles with forecasting method 2 for Scenario 3 applied to stationary time series

6.3.5.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,756585	0,784044	0,257	0,59924
0,843474	0,872191	0,87613	0,876896
0,744987	0,791774	0,798191	0,799439

Table 83 Error of type 3 for method 2 for Scenario 3 applied to stationary time series

6.3.6. Scenario 4– Optimization with IPOPT algorithm

6.3.6.i First-phase granules clusterization

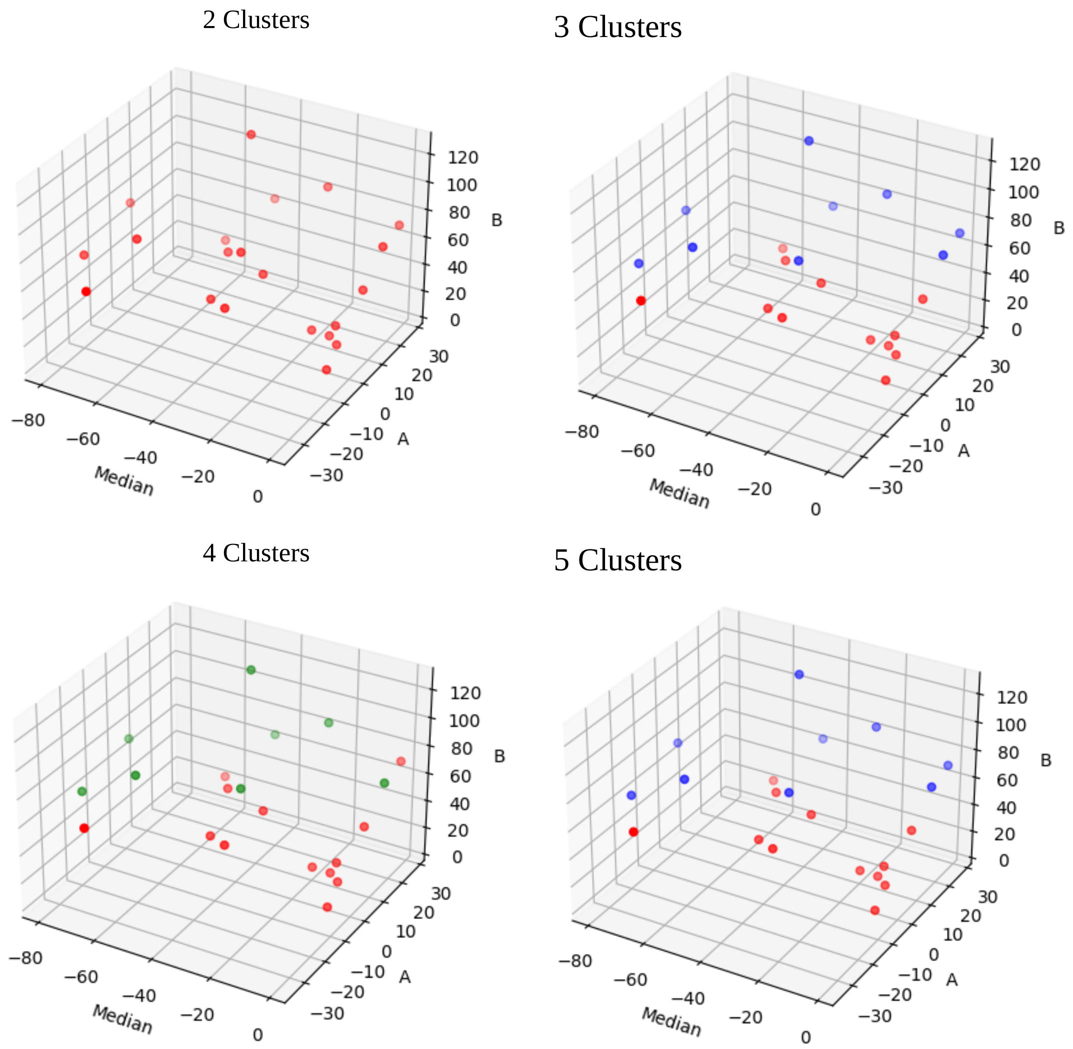


Figure 35 First-phase granules of stationary time series by applying Scenario 4

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.3.6.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	2,101795
Cluster 1	0,344814	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	0,000008	0,000014
Cluster 1	0,000011	0	0,000035
Cluster 2	6,088558	6,088558	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,00001	0,00001	0,00002
Cluster 1	6,088558	0	6,088558	6,088558
Cluster 2	0,000016	0,000021	0	0,000066
Cluster 3	6,088558	6,088558	6,088558	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,000014	0,000014	0,000027	0,000014
Cluster 1	0,00002	0	0,000025	0,000082	0,00002
Cluster 2	6,088558	6,088558	0	6,088558	6,088558
Cluster 3	6,088558	6,088558	6,088558	0	6,088558
Cluster 4	6,088558	6,088558	6,088558	6,088558	0

Table 84 Fuzzy cognitive map weights for Scenario 4 of stationary time series

6.3.6.iii Test membership values

Cluster 0	Cluster 1
0,701598	0,298402
0,701598	0,298402
0,701598	0,298402

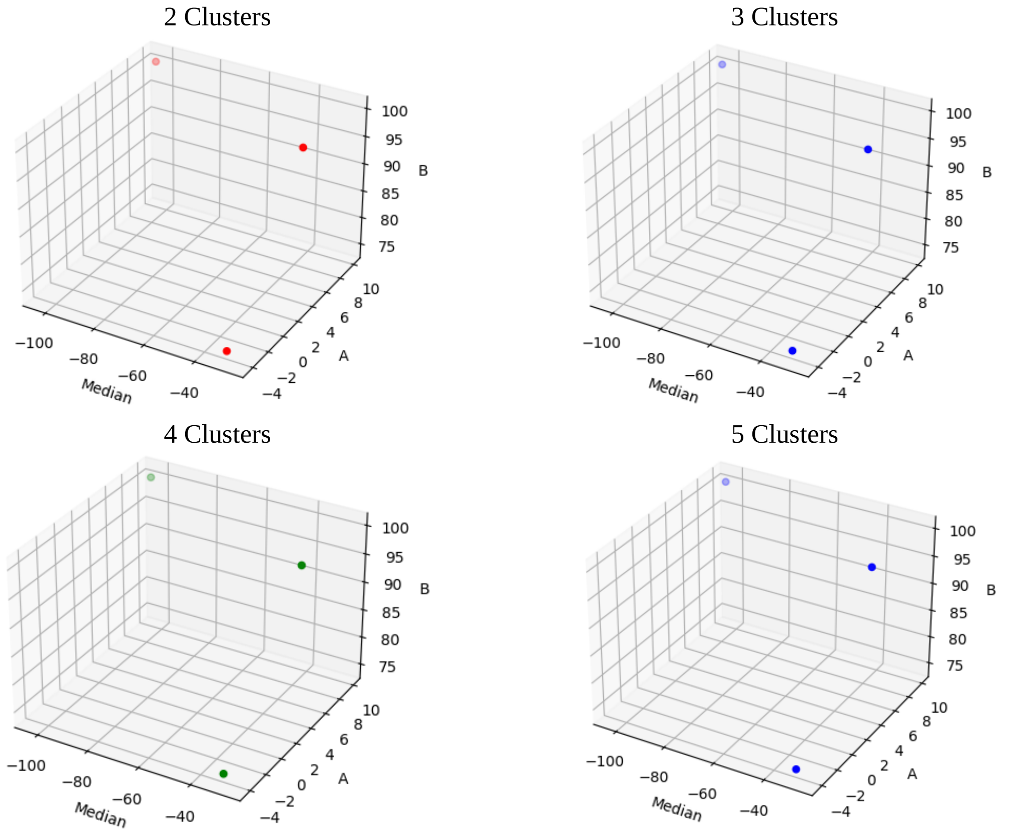
Cluster 0	Cluster 1	Cluster 2
0,423546	0,447386	0,129067
0,423546	0,447386	0,129067
0,423546	0,447386	0,129067

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,327553	0,256365	0,335843	0,080239
0,322084	0,252085	0,346932	0,078900
0,322084	0,252085	0,346932	0,078900

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,245161	0,259107	0,193715	0,059114	0,242903
0,224784	0,320688	0,177613	0,054201	0,222714
0,224704	0,320930	0,177550	0,054181	0,222634

Table 85 Membership degree between first and second-phase granules (clusters) for Scenario 4 of stationary time series

6.3.6.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 36 Test first-phase granules values and 3D representation of Scenario 4 applied to stationary time series

6.3.6.v Test membership forecast

Cluster 0	Cluster 1
0,6518495283	0,5601870647
0,7644797096	0,5559564068
0,7628749416	0,5655219443

Cluster 0	Cluster 1	Cluster 2
0,5000013094	0,5000022695	0,9950466417
0,5000043421	0,5000100934	0,9977365059
0,5000043512	0,5000101171	0,9977366552

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,5000018610	0,9895817535	0,5000039032	0,9963456403
0,5000085833	0,9999947375	0,5000235703	0,9999945163
0,5000086277	0,9999948540	0,5000236843	0,9999948540

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,5000028672	0,5000041671	0,9938074728	0,9969215181	0,9920108280
0,5000152585	0,5000342133	0,9999999875	0,9999999873	0,9999999876
0,5000153282	0,5000343556	0,9999999883	0,9999999883	0,9999999883

Table 86 Test membership degree forecast values for Scenario 4 applied to stationary time series

6.3.6.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,071007	0,758534	1,433545	2,203769
0,070289	0,763201	1,462879	2,282947
0,075108	0,763201	1,462880	2,283169

Table 87 Error of type 2(membership degree forecast) values for Scenario 4 applied to stationary time series

6.3.6.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-32,335	1,9417	52,312	Test granule 1	-28,591	1,6584	46,5
Test granule 2	-33,899	1,7755	53,997	Test granule 2	-28,567	1,6593	46,466
Test granule 3	-33,721	1,7944	53,805	Test granule 3	-28,567	1,6593	46,466
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-30,253	2,1485	49,857	Test granule 1	-27,985	5,8792	59,573
Test granule 2	-30,261	2,1433	49,841	Test granule 2	-27,943	5,8989	59,554
Test granule 3	-30,261	2,1433	49,841	Test granule 3	-27,943	5,8989	59,554

Table 88 Test fuzzy triangles with forecasting method 1 for Scenario 4 applied to stationary time series

6.3.6.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,271801	0,327096	0,2856	0,251534
0,580841	0,64151	0,615552	0,563431
0,422714	0,440376	0,423888	0,331374

Table 89 Error of type 3 for method 1 for Scenario 4 applied to stationary time series

6.3.6.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-33,59	2,1537	55,65	Test granule 1	-10,505	2,3332	21,234
Test granule 2	-33,59	2,1537	55,65	Test granule 2	-10,505	2,3332	21,234
Test granule 3	-33,59	2,1537	55,65	Test granule 3	-10,505	2,3332	21,234
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-8,9864	2,3064	17,727	Test granule 1	-8,965	2,143	16,93
Test granule 2	-39,998	0,5805	56,639	Test granule 2	-4,796	21,427	69,56
Test granule 3	-8,9864	2,3064	17,727	Test granule 3	-40,07	0,3681	55,47

Table 90 Test fuzzy triangles with forecasting method 2 for Scenario 4 applied to stationary time series

6.3.6.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,254661	0,730568	0,77637	0,782371
0,572787	0,840732	0,545564	0,65966
0,404968	0,740521	0,778998	0,469286

Table 91 Error of type 3 for method 2 for Scenario 4 applied to stationary time series

6.3.7. Scenario 5– Optimization with IPOPT algorithm

6.3.7.i First-phase granules clusterization

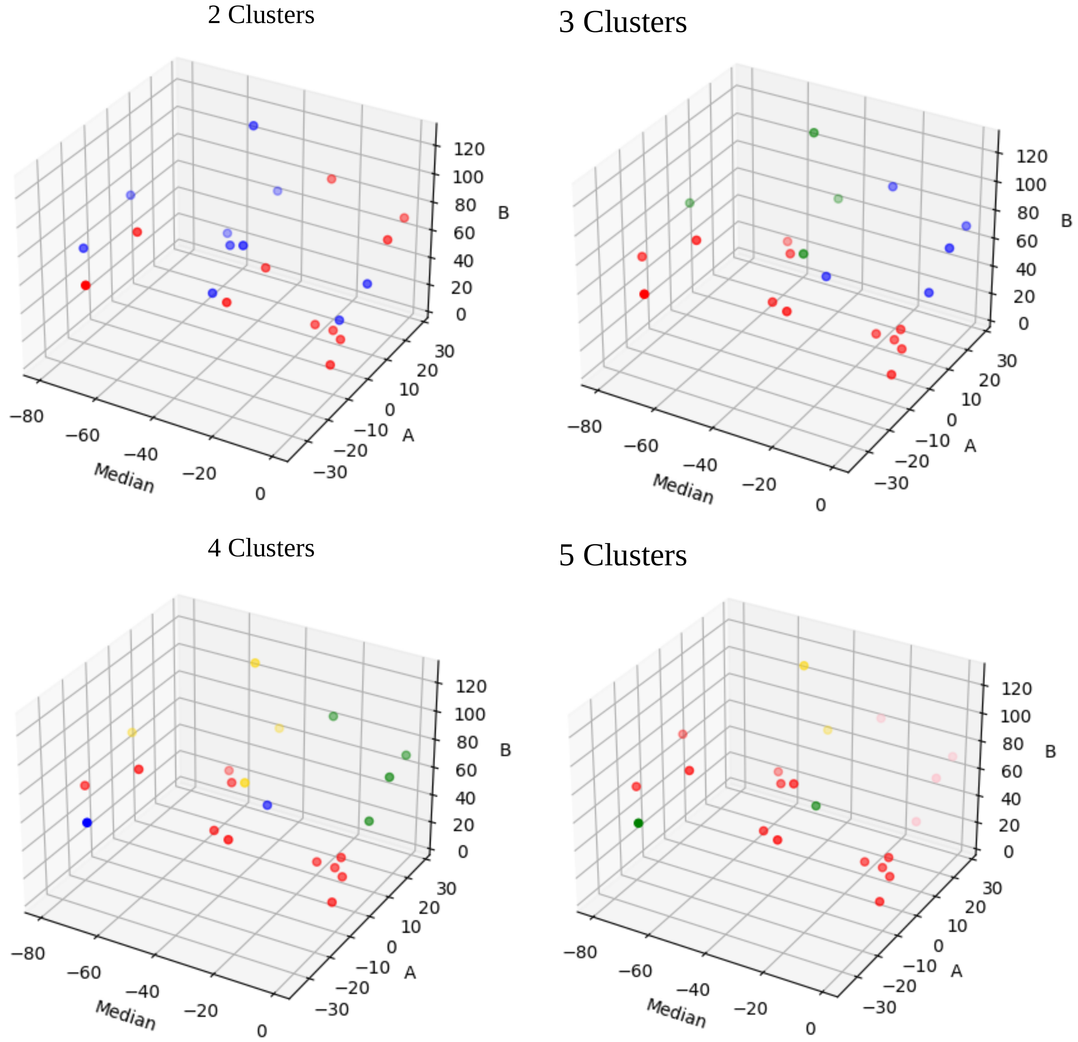


Figure 37 First-phase granules of stationary time series by applying Scenario 5

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.3.7.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	-0,431685
Cluster 1	0,000008	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	35,474317	0,00119
Cluster 1	0,000086	0	0,000177
Cluster 2	1,247644	20,277968	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	2,443713	7,982287	20,432044
Cluster 1	0,527651	0	3,423321	0,893781
Cluster 2	0,527122	1,906181	0	2,909127
Cluster 3	1,618823	0,030158	0,011928	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	5,028419	5,291671	1,485877	2,524841
Cluster 1	1,318118	0	1,318118	1,318118	1,318118
Cluster 2	0,750701	2,831164	0	3,773397	3,505982
Cluster 3	0,69332	0,045866	0,046173	0	0,027975
Cluster 4	0,06463	0,761425	3,341074	5,864863	0

Table 92 Fuzzy cognitive map weights for Scenario 5 of stationary time series

6.3.7.iii Test membership values

Cluster 0	Cluster 1
1,215976	-0,215976
0,433230	0,566770
0,775301	0,224699

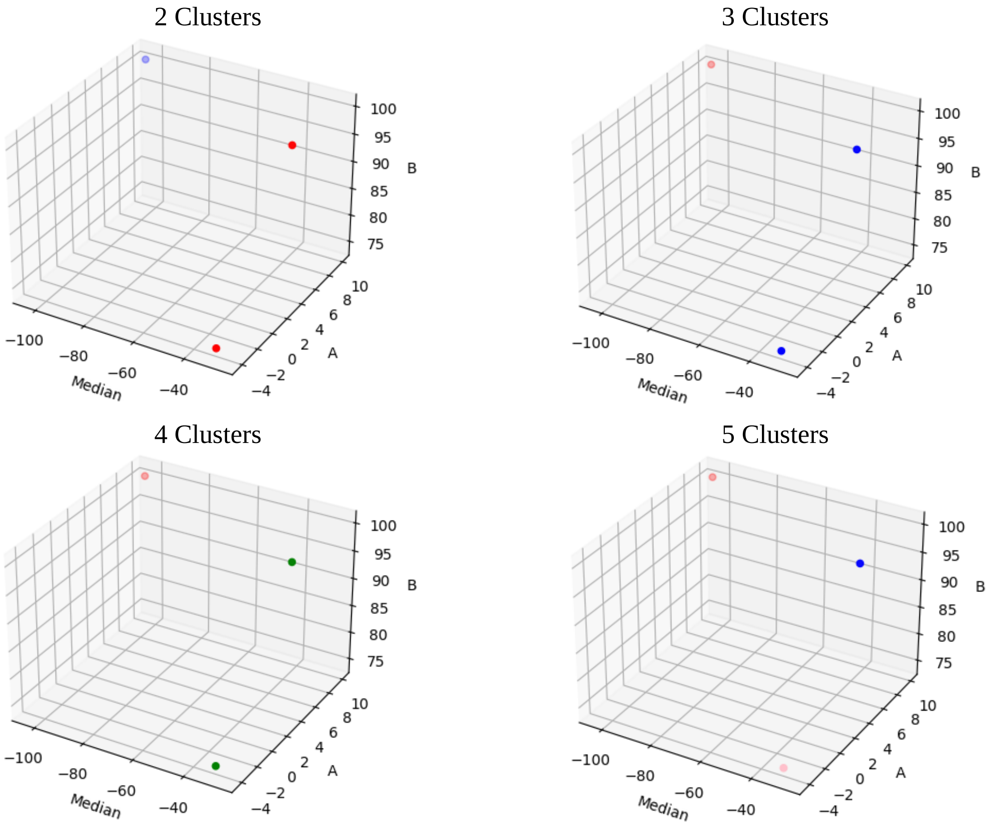
Cluster 0	Cluster 1	Cluster 2
-0,868312	2,767028	-0,898717
0,498879	0,112939	0,388182
-0,235785	1,319685	-0,083900

Cluster 0	Cluster 1	Cluster 2	Cluster 3
-1,988968	1,749745	2,820197	-1,580975
0,414039	0,237508	0,052420	0,296033
-0,329429	0,450707	1,025890	-0,147168

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
-2,735882	1,456605	1,575237	-1,649721	2,353761
0,435900	-0,022771	0,232241	0,284032	0,070598
-0,269966	0,645236	0,244689	-0,099343	0,479384

Table 93 Membership degree between first and second-phase granules (clusters) for Scenario 5 of stationary time series

6.3.7.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 38 Test first-phase granules values and 3D representation of pr ocess 5 applied to stationary time series

6.3.7.v Test membership forecast

Cluster 0	Cluster 1
0,4283170739	0,5000006832
0,4462478128	0,5000008836
0,4462477914	0,5000009206

Cluster 0	Cluster 1	Cluster 2
0,9815810172	0,5000294255	0,9432900403
0,9999999802	0,5000627613	0,9999883971
0,9999999803	0,5000656652	0,9999886684

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,9997612435	0,6654001390	0,8424439816	0,6424178127
0,9999999995	0,9817587765	0,9750166339	0,8386952819
1,0000000000	0,9901954457	0,9921425370	0,8402461548

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,7551608645	0,8341406757	0,8279138693	0,5795467499	0,9587136976
0,9999929126	0,9839250593	0,9997917539	0,6517803038	0,9989406348
0,9999989094	0,9919323865	0,9999250071	0,6926531108	0,9996567799

Table 94 Test membership degree forecast values for Scenario 5 applied to stationary time series

6.3.7.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,133030	11,954376	18,963287	20,049126
0,004628	0,775294	2,042927	2,917830
0,184067	3,373756	3,034555	3,201332

Table 95 Error of type 2(membership degree forecast) values for Scenario 5 applied to stationary time series

6.3.7.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	-29,429	2,2502	49,184	Test granule 1	-36,27	-1,116	51,643
Test granule 2	-29,817	2,209	49,602	Test granule 2	-35,899	-1,1195	51,087
Test granule 3	-29,817	2,209	49,602	Test granule 3	-35,899	-1,1195	51,087
4 clusters	**a**	**m**	**b**	**5 clusters**	**a**	**m**	**b**
Test granule 1	-36,155	1,7611	59,394	Test granule 1	-33,878	5,7672	70,146
Test granule 2	-34,27	2,3842	57,658	Test granule 2	-35,788	4,7971	70,456
Test granule 3	-34,178	2,4665	57,731	Test granule 3	-35,565	4,795	70,077

Table 96 Test fuzzy triangles with forecasting method 1 for Scenario 5 applied to stationary time series

6.3.7.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,300245	0,285967	0,240255	0,157581
0,618439	0,598627	0,559145	0,480949
0,421837	0,48219	0,391179	0,300873

Table 97 Error of type 3 for method 1 for Scenario 5 applied to stationary time series

6.3.7.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	-40,182	3,4553	60,222	Test granule 1	-37,797	-6,2161	42,27
Test granule 2	-40,182	3,4553	60,222	Test granule 2	-37,797	-6,2161	42,27
Test granule 3	-40,182	3,4553	60,222	Test granule 3	-37,797	-6,2161	42,27
4 clusters	**a**	**m**	**b**	**5 clusters**	**a**	**m**	**b**
Test granule 1	-35,86	-4,025	40,515	Test granule 1	-6,86	20,2	68,75
Test granule 2	-35,86	-4,025	40,515	Test granule 2	-39,03	-3,67	44,94
Test granule 3	-35,86	-4,025	40,515	Test granule 3	-39,03	-3,67	44,94

Table 98 Test fuzzy triangles with forecasting method 2 for Scenario 5 applied to stationary time series

6.3.7.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,285096	0,375697	0,357527	0,492337
0,514519	0,654788	0,661803	0,624609
0,410439	0,602647	0,584575	0,572314

Table 99 Error of type 3 for method 2 for Scenario 5 applied to stationary time series

6.3.8. Scenario 6– Optimization with IPOPT algorithm

6.3.8.i First-phase granules clusterization

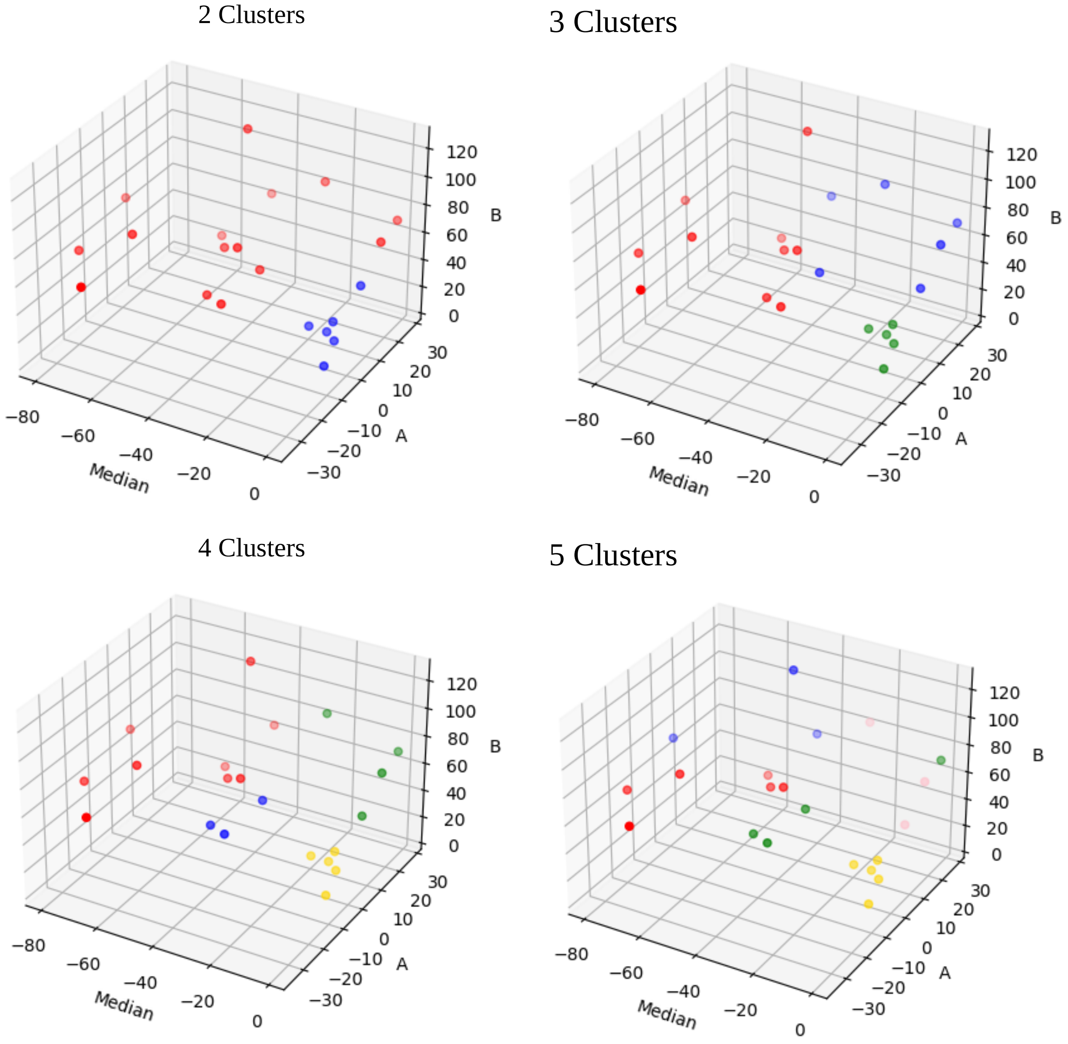


Figure 39 First-phase granules of stationary time series by applying Scenario 6

Where:

Cluster 0
Cluster 1
Cluster 2
Cluster 3
Cluster 4

6.3.8.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	3,748577
Cluster 1	6,801842	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	0,000012	0,000015
Cluster 1	0,000019	0	0,000023
Cluster 2	0,000031	0,000025	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,000017	0,000019	0,000022
Cluster 1	0,000047	0	0,000053	0,000055
Cluster 2	0,000041	0,000034	0	0,000042
Cluster 3	0,000037	0,000029	0,000032	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,00003	0,000029	0,000034	0,000033
Cluster 1	0,000088	0	0,000089	0,000104	0,000093
Cluster 2	0,000044	0,000051	0	0,000048	0,000046
Cluster 3	0,00004	0,000046	0,000034	0	0,000036
Cluster 4	0,000066	0,000062	0,000058	0,000065	0

Table 100 Fuzzy cognitive map weights for Scenario 6 of stationary time series

6.3.8.iii Test membership values

Cluster 0	Cluster 1
0,709147	0,290853
0,651008	0,348992
0,657878	0,342122

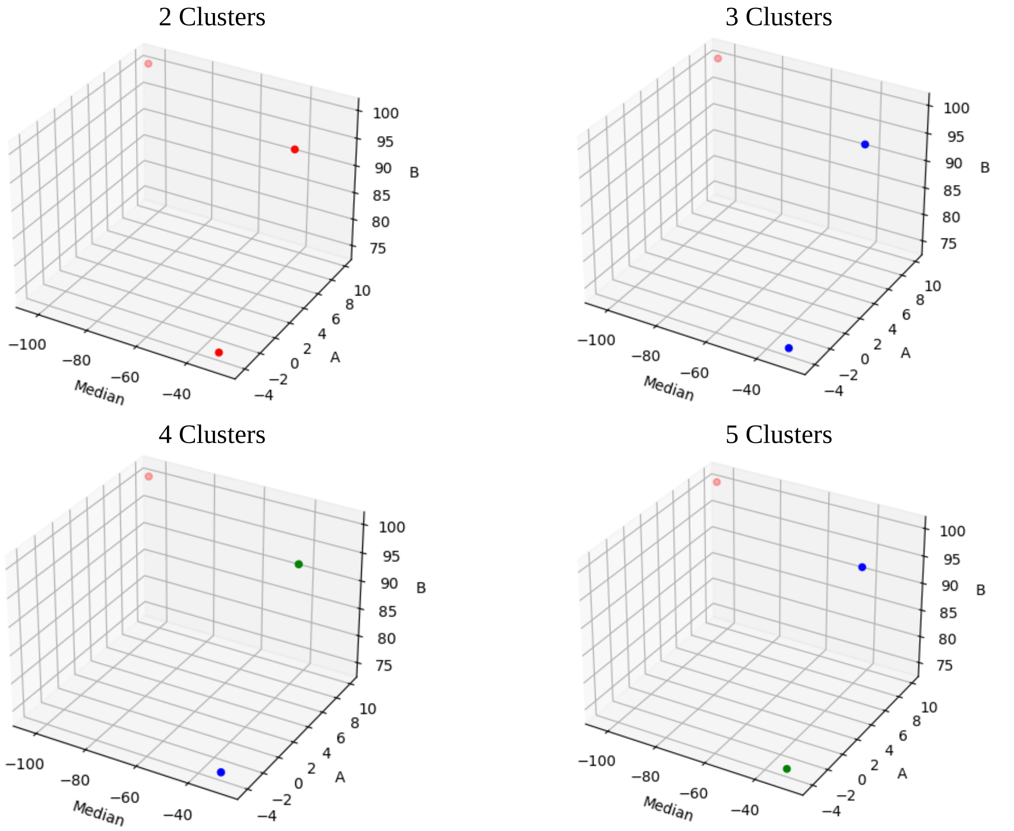
Cluster 0	Cluster 1	Cluster 2
0,339496	0,416353	0,244152
0,395122	0,343181	0,261697
0,339438	0,411357	0,249205

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,244364	0,289048	0,265156	0,201431
0,285487	0,258719	0,243165	0,212630
0,248872	0,263054	0,284767	0,203307

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,200873	0,187651	0,225218	0,178767	0,207492
0,219631	0,212609	0,203760	0,175471	0,188529
0,195262	0,220789	0,205257	0,168502	0,210191

Table 101 Membership degree between first and second-phase granules (clusters) for Scenario 6 of stationary time series

6.3.8.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-30,72	-4,22	74,20
Test granule 2	-103,83	9,94	100,16
Test granule 3	-26,96	4,08	100,27

Figure 40 Test first-phase granules values and 3D representation of pr ocess 6 applied to stationary time series

6.3.8.v Test membership forecast

Cluster 0	Cluster 1
0,7874592877	0,9881723084
0,9759726004	0,9953032348
0,9765915289	0,9986926060

Cluster 0	Cluster 1	Cluster 2
0,5000020586	0,5000033557	0,5000051531
0,5000034254	0,5000052482	0,5000069246
0,5000034255	0,5000052482	0,5000069246

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,5000034481	0,5000095775	0,5000073034	0,5000064054
0,5000072611	0,5000194863	0,5000146228	0,5000121290
0,5000072612	0,5000194865	0,5000146230	0,5000121292

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,5000061569	0,5000180683	0,5000094606	0,5000081079	0,5000127373
0,5000157850	0,5000466920	0,5000236632	0,5000194595	0,5000314513
0,5000157856	0,5000466933	0,5000236641	0,5000194602	0,5000314523

Table 102 Test membership degree forecast values for Scenario 6 applied to stationary time series

6.3.8.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,492386	0,098221	0,254158	0,451329
0,523320	0,092386	0,252806	0,451374
0,532664	0,096541	0,253587	0,451659

Table 103 Error of type 2(membership degree forecast) values for Scenario 6 applied to stationary time series

6.3.8.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-28,748	2,3225	48,451	Test granule 1	-34,56	1,4357	54,839
Test granule 2	-30,711	2,1141	50,564	Test granule 2	-34,56	1,4357	54,839
Test granule 3	-30,685	2,1169	50,535	Test granule 3	-34,56	1,4357	54,839
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-33,145	2,4931	56,17	Test granule 1	-33,942	4,6507	66,894
Test granule 2	-33,145	2,4932	56,17	Test granule 2	-33,942	4,6509	66,895
Test granule 3	-33,145	2,4932	56,17	Test granule 3	-33,942	4,6509	66,895

Table 104 Test fuzzy triangles with forecasting method 1 for Scenario 6 applied to stationary time series

6.3.8.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,313865	0,264401	0,248927	0,177734
0,610209	0,575405	0,570852	0,506909
0,422032	0,42396	0,394993	0,311655

Table 105 Error of type 3 for method 1 for Scenario 6 applied to stationary time series

6.3.8.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-8,335	3,4597	21,691	Test granule 1	-8,9778	1,8864	16,478
Test granule 2	-8,335	3,4597	21,691	Test granule 2	-8,9778	1,8864	16,478
Test granule 3	-8,335	3,4597	21,691	Test granule 3	-8,9778	1,8864	16,478
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-37,093	-4,8874	47,802	Test granule 1	-52,53	13,171	100,4
Test granule 2	-37,093	-4,8874	47,802	Test granule 2	-52,53	13,171	100,4
Test granule 3	-37,093	-4,8874	47,802	Test granule 3	-52,53	13,171	100,4

Table 106 Test fuzzy triangles with forecasting method 2 for Scenario 6 applied to stationary time series

6.3.8.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,756584	0,784047	0,306376	0,599137
0,843472	0,872193	0,626008	0,256002
0,744985	0,791777	0,54503	0,264868

Table 107 Error of type 3 for method 2 for Scenario 6 applied to stationary time series

6.3.9. Summary tables

6.3.9.i Cluster forecast tables

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	Cluster 0	Cluster 1	Cluster 1
Test granule 2	Cluster 0	Cluster 1	Cluster 1
Test granule 3	Cluster 0	Cluster 1	Cluster 1
	3 CLUSTERS		
Test granule 1	Cluster 2	Cluster 1	Cluster 2
Test granule 2	Cluster 2	Cluster 0	Cluster 2
Test granule 3	Cluster 2	Cluster 0	Cluster 2
	4 CLUSTERS		
Test granule 1	Cluster 3	Cluster 0	Cluster 0
Test granule 2	Cluster 3	Cluster 0	Cluster 1
Test granule 3	Cluster 3	Cluster 0	Cluster 2
	5 CLUSTERS		
Test granule 1	Cluster 2	Cluster 0	Cluster 1
Test granule 2	Cluster 2	Cluster 0	Cluster 3
Test granule 3	Cluster 2	Cluster 0	Cluster 3

Table 108 Second-phase granules forecast accuracy for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
	2 CLUSTERS		
Test granule 1	Cluster 0	Cluster 0	Cluster 1
Test granule 2	Cluster 0	Cluster 1	Cluster 1
Test granule 3	Cluster 0	Cluster 0	Cluster 1
	3 CLUSTERS		
Test granule 1	Cluster 2	Cluster 0	Cluster 2
Test granule 2	Cluster 2	Cluster 0	Cluster 2
Test granule 3	Cluster 2	Cluster 0	Cluster 2
	4 CLUSTERS		
Test granule 1	Cluster 3	Cluster 0	Cluster 1
Test granule 2	Cluster 1	Cluster 0	Cluster 1
Test granule 3	Cluster 1/3	Cluster 0	Cluster 1
	5 CLUSTERS		
Test granule 1	Cluster 3	**Cluster 4**	Cluster 1
Test granule 2	Cluster 4	**Cluster 0**	Cluster 1
Test granule 3	Cluster 2/3/4	**Cluster 0**	Cluster 1

Table 109 Second-phase granules forecast accuracy for Scenarios optimized by IPOPT algorithm of the stationary time series

6.3.9.ii Error type 2 tables

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	0,027419	1,132805	0,485258
Test granule 2	0,036543	0,004821	0,434316
Test granule 3	0,027165	0,183543	0,456329
	3 CLUSTERS		
Test granule 1	0,362202	9,254288	0,055580
Test granule 2	0,415635	0,390972	0,039547
Test granule 3	0,333091	2,537957	0,047841
	4 CLUSTERS		
Test granule 1	0,467926	18,893930	0,010791
Test granule 2	0,574787	0,989967	0,006295
Test granule 3	0,567610	2,092913	0,011785
	5 CLUSTERS		
Test granule 1	0,692287	20,859655	0,005585
Test granule 2	1,088974	1,725956	0,002410
Test granule 3	1,019136	1,968028	0,006410

Table 110 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
	2 CLUSTERS		
Test granule 1	0,071007	1,133030	0,492386
Test granule 2	0,070289	0,004628	0,523320
Test granule 3	0,075108	0,184067	0,532664
	3 CLUSTERS		
Test granule 1	0,758534	11,954376	0,098221
Test granule 2	0,763201	0,775294	0,092386
Test granule 3	0,763201	3,373756	0,096541
	4 CLUSTERS		
Test granule 1	1,433545	18,963287	0,254158
Test granule 2	1,462879	2,042927	0,252806
Test granule 3	1,462880	3,034555	0,253587
	5 CLUSTERS		
Test granule 1	2,203769	**20,049126**	0,451329
Test granule 2	2,282947	**2,917830**	0,451374
Test granule 3	2,283169	**3,201332**	0,451659

Table 111 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the stationary time series

6.3.9.iii Error type 3 tables (Method 1)

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,253650357	0,300284039	0,351906725
Test granule 2	0,508571495	0,618280746	0,622990698
Test granule 3	0,424327165	0,421879522	0,421740157
3 CLUSTERS			
Test granule 1	0,291861829	0,266089426	0,273281107
Test granule 2	0,668916935	0,532192306	0,594212125
Test granule 3	0,440850659	0,484511461	0,43148732
4 CLUSTERS			
Test granule 1	0,367090892	0,24071926	0,257549936
Test granule 2	0,651213948	0,465635956	0,581581826
Test granule 3	0,437432793	0,444697281	0,407641459
5 CLUSTERS			
Test granule 1	0,202193513	0,162264065	0,176498992
Test granule 2	0,634511756	0,426584864	0,509219838
Test granule 3	0,467629274	0,357463255	0,313780367

Table 112 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,271804105	0,300322522	0,31394081
Test granule 2	0,58083719	0,618435937	0,610203805
Test granule 3	0,422752266	0,421871906	0,422078288
3 CLUSTERS			
Test granule 1	0,327170607	0,285972146	0,26440077
Test granule 2	0,641507502	0,598623381	0,575399897
Test granule 3	0,440421141	0,482229076	0,423998934
4 CLUSTERS			
Test granule 1	0,285674363	0,240261132	0,248928899
Test granule 2	0,615550414	0,55914458	0,570849343
Test granule 3	0,42393344	0,391215444	0,395033128
5 CLUSTERS			
Test granule 1	0,25161444	**0,157588419**	0,177741433
Test granule 2	0,563427757	**0,480946571**	0,506904595
Test granule 3	0,331413449	**0,300913868**	0,311689133

Table 113 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the stationary time series

6.3.9.iv Error type 3 tables (Method 2)

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	0,254660986	0,28509529	0,75658512
Test granule 2	0,572786893	0,514517365	0,843473566
Test granule 3	0,404967634	0,410438657	0,744987434
3 CLUSTERS			
Test granule 1	0,730568078	0,375692688	0,784044347
Test granule 2	0,840732015	0,654789417	0,872190904
Test granule 3	0,740520905	0,602643363	0,791773665
4 CLUSTERS			
Test granule 1	0,7763698	0,357522448	0,257000183
Test granule 2	0,864349086	0,661805263	0,876129575
Test granule 3	0,778997791	0,584570863	0,798190541
5 CLUSTERS			
Test granule 1	0,2992954	0,349276159	0,599239509
Test granule 2	0,551679989	0,624643575	0,876895714
Test granule 3	0,469262813	0,572309801	0,799438733

Table 114 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by genetic algorithm of the stationary time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	0,254660986	0,28509529	0,75658512
Test granule 2	0,572786893	0,514517365	0,843473566
Test granule 3	0,404967634	0,410438657	0,744987434
3 CLUSTERS			
Test granule 1	0,730568078	0,375692688	0,784044347
Test granule 2	0,840732015	0,654789417	0,872190904
Test granule 3	0,740520905	0,602643363	0,791773665
4 CLUSTERS			
Test granule 1	0,7763698	0,357522448	0,306380795
Test granule 2	0,54556397	0,661805263	0,626010377
Test granule 3	0,778997791	0,584570863	0,545033889
5 CLUSTERS			
Test granule 1	0,782371266	**0,492303463**	0,599239509
Test granule 2	0,659659702	**0,624643575**	0,256022998
Test granule 3	0,46928566	**0,572309801**	0,264953164

Table 115 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the stationary time series

6.4. Time series with seasonality

6.4.1. Errors of Fuzzy C-means

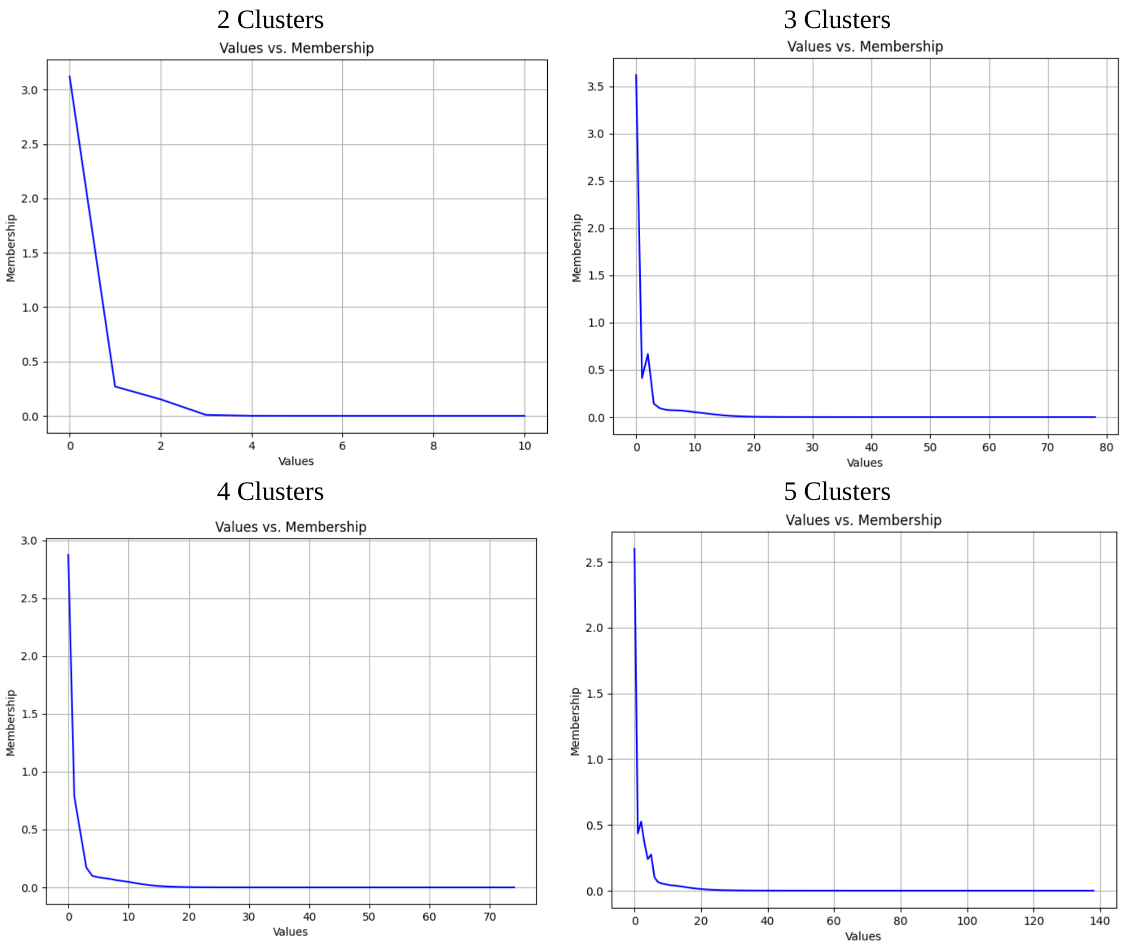


Figure 41 Seasonality time series decaying error (Y axis) of the Fuzzy C-means clustering as iterations go by (X axis)

6.4.2. First-phase granules (blue) and second-phase granules (red)

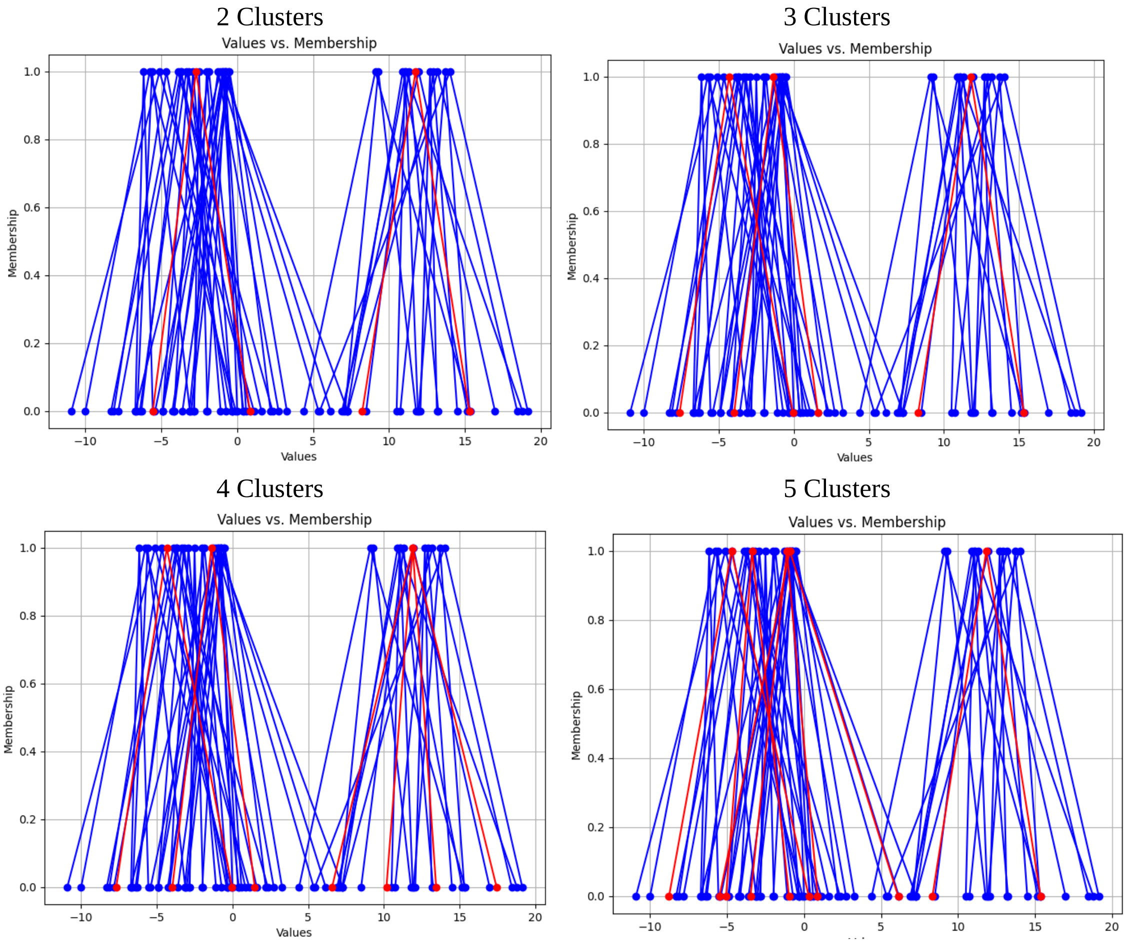


Figure 42 Representation as fuzzy triangles of first-phase granules(blue) and second-phase granules (red) for the seasonality time series

6.4.3. Scenario 1 – Optimization with genetic algorithm

6.4.3.i First-phase granules clusterization

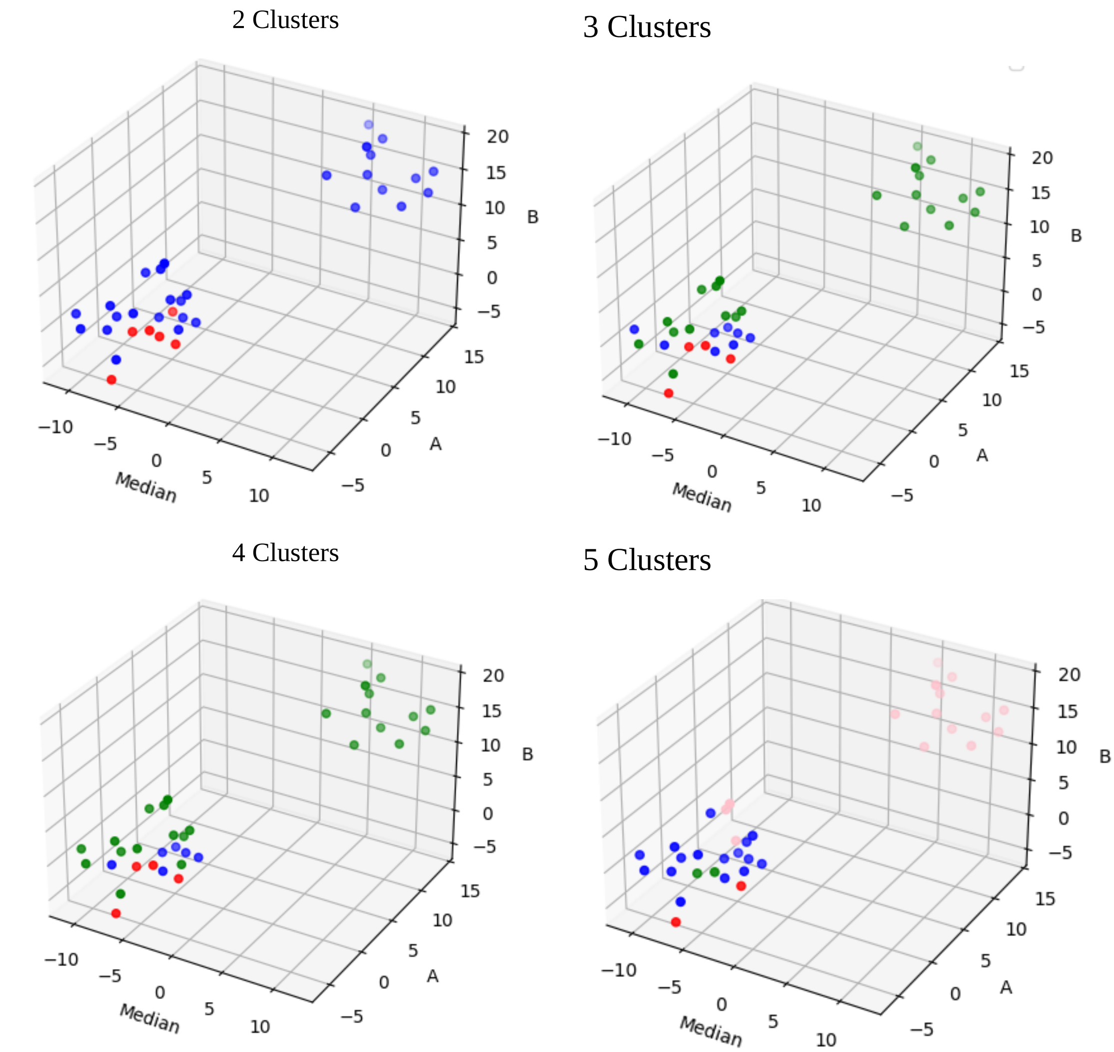


Figure 43 First-phase granules of seasonality time series by applying Scenario 1

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.3.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	4,574949
Cluster 1	17,327478	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	4,564441	0,118535
Cluster 1	-9,777234	0	2,222007
Cluster 2	-1,922507	4,320167	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,459699	-1,017791	0,022813
Cluster 1	6,368079	0	0,214691	0,076292
Cluster 2	-2,986584	0,407461	0	-1,12649
Cluster 3	-0,788085	-1,794426	1,074117	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	1,788463	1,137528	-0,21341	-0,252854
Cluster 1	0,091579	0	0,50043	0,977413	0,310982
Cluster 2	0,716681	1,142091	0	0,887213	1,197291
Cluster 3	1,376928	-0,225228	-2,211087	0	-0,703461
Cluster 4	0,009183	0,081822	1,980862	-0,043941	0

Table 116 Fuzzy cognitive map weights for Scenario 1 of seasonality time series

6.4.3.iii Test membership values

Cluster 0	Cluster 1
0,258993	0,741007
0,499816	0,500184
0,055690	0,944310

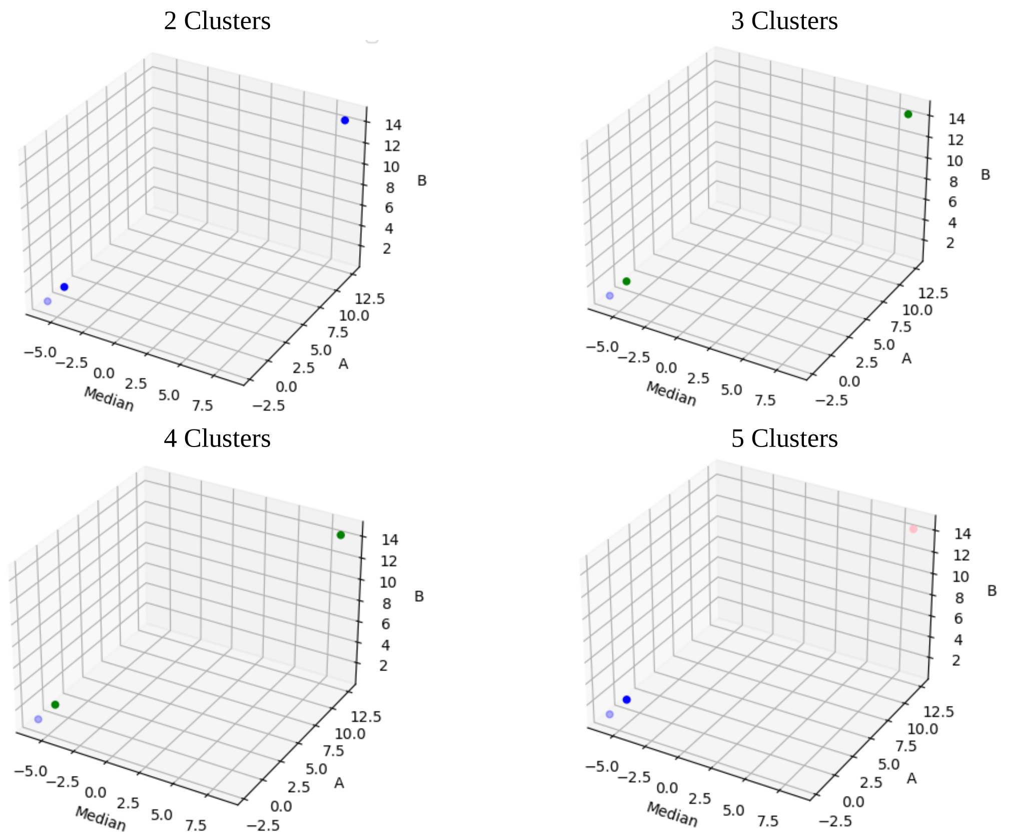
Cluster 0	Cluster 1	Cluster 2
-0,014897	0,404014	0,610883
-0,036118	0,518059	0,518059
-0,003715	0,100763	0,902952

Cluster 0	Cluster 1	Cluster 2	Cluster 3
-0,003954	0,234479	0,384738	0,384738
-0,009892	0,336631	0,336631	0,336631
-0,000851	0,050472	0,490813	0,459566

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,060933	0,477314	-0,191161	0,175600	0,477314
0,164406	0,450457	-0,515777	0,450457	0,450457
0,014939	0,295337	-0,046867	0,043052	0,693539

Table 117 Membership degree between first and second-phase granules (clusters) for Scenario 1 of seasonality time series

Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 44 Test first-phase granules values and 3D representation of pr ocess 1 applied to seasonality time series

6.4.3.iv Test membership forecast

Cluster 0	Cluster 1
0,986653	0,736915
0,966797	1,000000
0,989798	1,000000

Cluster 0	Cluster 1	Cluster 2
0,644687	0,884031	0,615592
0,983827	0,007136	0,929547
0,535629	0,000524	0,134635

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,389861	0,533052	0,375107	0,602612
0,469291	0,931450	0,164380	0,297160
0,566519	0,954629	0,204761	0,134157

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,578531	0,557744	0,771625	0,397255	0,481546
0,841404	0,726544	0,878747	0,201990	0,826637
0,885631	0,725379	0,930973	0,178055	0,858024

Table 118 Test membership degree forecast values for Scenario 1 applied to seasonality time series

6.4.3.v Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,529505	0,665490	0,291799	1,250482
0,467887	1,470650	0,614654	2,682495
0,875659	0,891251	1,327124	1,944493

Table 119 Error of type 2(membership degree forecast) values for Scenario 1 applied to seasonality time series

6.4.3.vi Test fuzzy triangle forecasted values (Method 1)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	0,32759	3,455089	7,000774	Test granule 1	-1,5424	1,5323	5,0423
Test granule 2	1,420608	4,60008	8,145445	Test granule 2	0,1024	3,4934	7,393
Test granule 3	1,365772	4,542636	8,088018	Test granule 3	-4,4095	-1,0757	3,0299

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	1,8363	4,8426	8,1134	Test granule 1	-3,3672	-0,2037	3,7991
Test granule 2	-1,7116	1,1632	4,4169	Test granule 2	-2,761	0,5695	4,811
Test granule 3	-2,9118	0,1364	3,6395	Test granule 3	-2,7655	0,5574	4,7893

Table 120 Test fuzzy triangles with forecasting method 1 for Scenario 1 applied to seasonality time series

6.4.3.vii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,658919	1,021308	2,103359	0,488714
2,388211	2,034388	1,241481	1,102394
0,606289	1,069066	0,975693	0,927392

Table 121 Error of type 3 for method 1 for Scenario 1 applied to seasonality time series

6.4.3.viii Test fuzzy triangle forecasted values (Method 2)

2 clusters	a	m	b	3 clusters	a	m	b
Test granule 1	-4,9519	-3,1286	-1,1632	Test granule 1	-4,9195	-2,2611	0,2568
Test granule 2	0,2869	3,6732	7,6266	Test granule 2	-5,6633	-3,9618	-1,7121
Test granule 3	0,2869	3,6732	7,6266	Test granule 3	-5,6633	-3,9618	-1,7121

4 clusters	a	m	b	5 clusters	a	m	b
Test granule 1	10,164	11,899	13,446	Test granule 1	-5,418	-3,404	-0,658
Test granule 2	-4,3589	-1,8983	0,1996	Test granule 2	-5,418	-3,404	-0,658
Test granule 3	-4,3589	-1,8983	0,1996	Test granule 3	-5,418	-3,404	-0,658

Table 122 Test fuzzy triangles with forecasting method 2 for Scenario 1 applied to seasonality time series

6.4.3.ix Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,561386	0,304191	4,400111	0,587583
2,100969	0,511203	0,28098	0,35902
0,674073	1,318909	1,170397	1,266667

Table 123 Error of type 3 for method 2 for Scenario 1 applied to seasonality time series

6.4.4. Scenario 2– Optimization with genetic algorithm

6.4.4.i First-phase granules clusterization

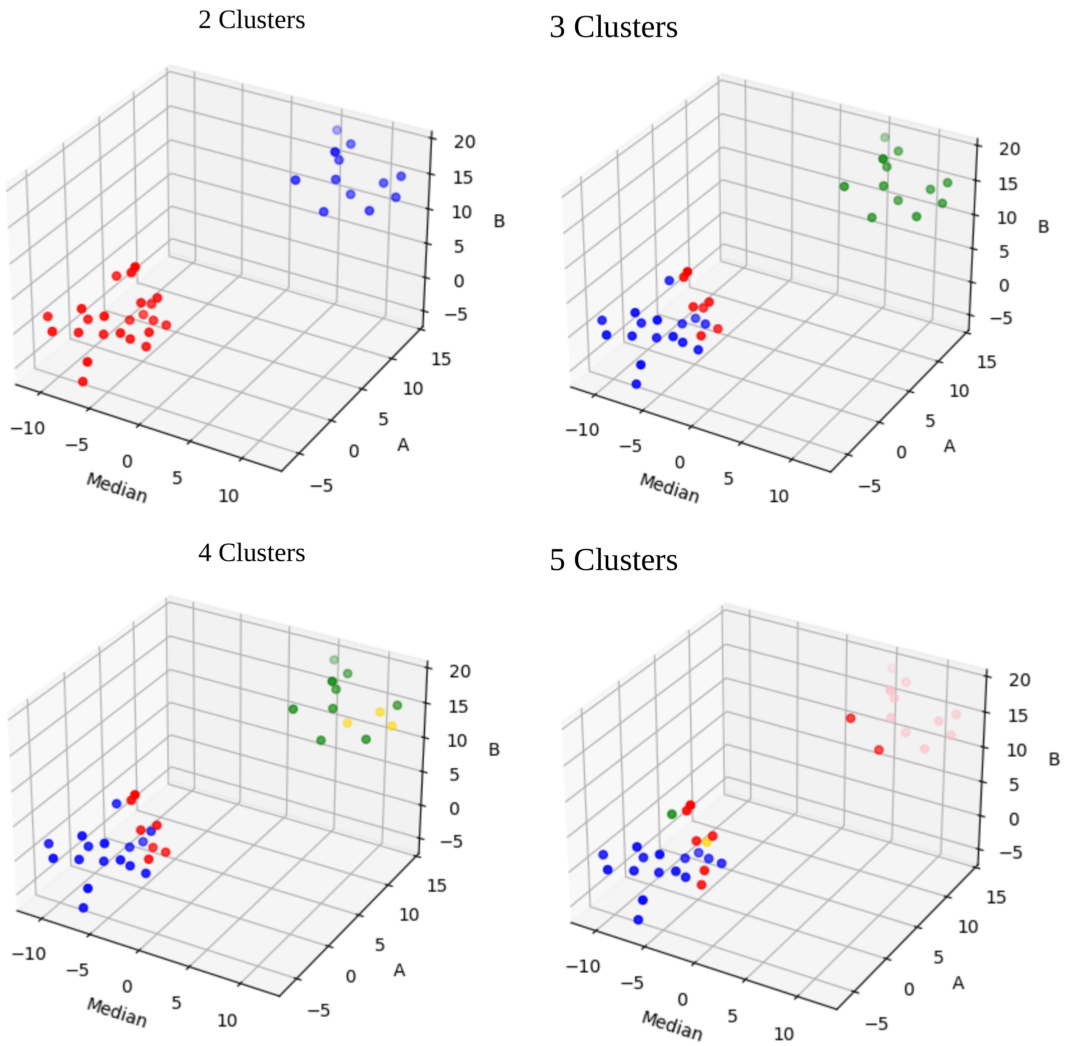


Figure 45 First-phase granules of seasonality time series by applying Scenario 2

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.4.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	-20,329095
Cluster 1	-15,922169	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	13,494414	-0,975114
Cluster 1	4,708194	0	-4,631698
Cluster 2	-1,388644	0,345368	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	2,330857	3,998571	1,436544
Cluster 1	0,982039	0	-6,071403	-6,617794
Cluster 2	-0,705636	0,554444	0	1,887705
Cluster 3	-2,408199	-1,052482	0,091272	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-0,637641	10,279521	3,472115	0,625468
Cluster 1	5,095433	0	8,936499	-0,42451	1,593762
Cluster 2	1,677846	3,481024	0	-0,395177	3,553297
Cluster 3	0,973216	-0,48476	0,658073	0	-0,399127
Cluster 4	-1,004658	-2,35883	0,499228	0,433087	0

Table 124 Fuzzy cognitive map weights for Scenario 2 of seasonality time series

6.4.4.iii Test membership values

Cluster 0	Cluster 1
1,071596	-0,071596
1,224016	-0,224016
-0,283927	1,283927

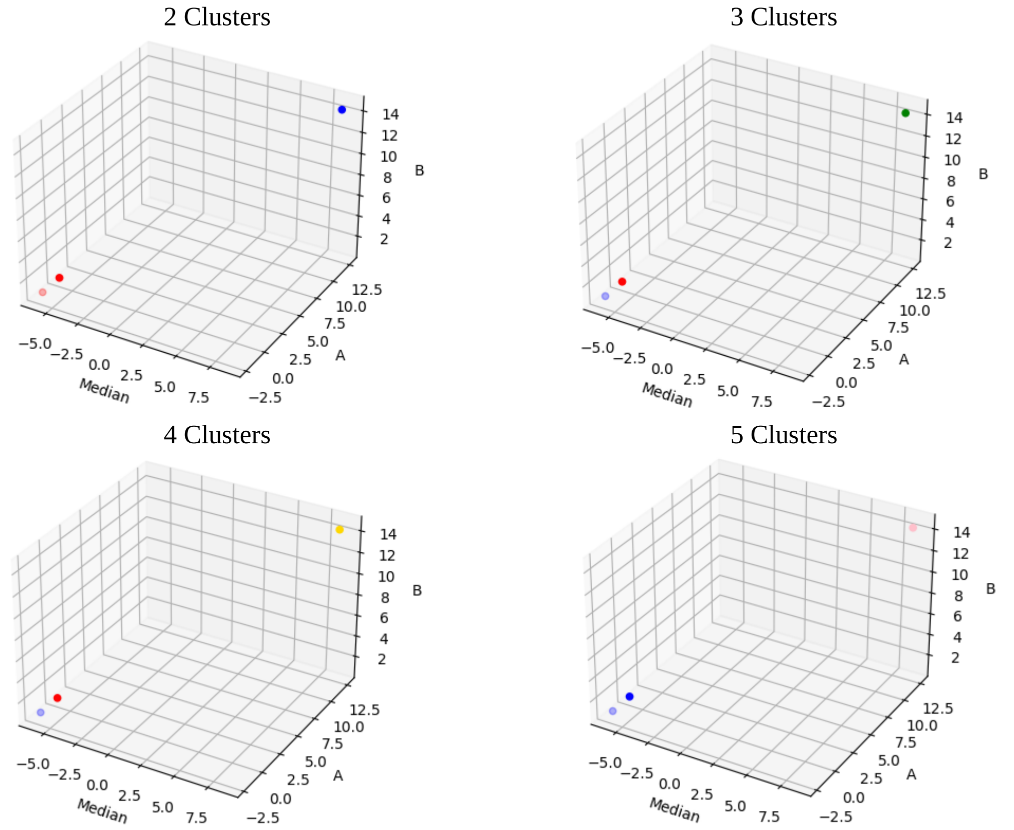
Cluster 0	Cluster 1	Cluster 2
0,640561	0,385815	-0,026376
0,484091	0,576417	-0,060508
-0,670914	-0,208293	1,879207

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,654939	0,398305	-0,026475	-0,026769
0,528928	0,601598	-0,064903	-0,065623
-0,256293	-0,082580	0,644113	0,694760

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
-0,745199	2,877425	-0,532052	-0,628019	0,027845
0,057833	0,834210	0,051270	0,063247	-0,006560
-1,852804	-0,039362	-1,322852	-0,519181	4,734200

Table 125 Membership degree between first and second-phase granules (clusters) for Scenario 2 of seasonality time series

6.4.4.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 46 Test first-phase granules values and 3D representation of pr ocess 2 applied to seasonality time series

6.4.4.v Test membership forecast

Cluster 0	Cluster 1
0,000000	0,996454
0,000000	0,500000
0,000039	0,500000

Cluster 0	Cluster 1	Cluster 2
0,002023	0,000000	0,772918
0,320020	0,027374	0,499298
0,470665	0,308769	0,392947

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,972216	0,000101	0,818187	0,710755
0,986518	0,000164	0,658299	0,093917
0,940897	0,025345	0,373146	0,089821

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,000000	0,000000	1,000000	0,000000	0,990866
0,999982	0,999973	0,971275	0,565273	0,622278
0,999992	1,000000	0,999213	0,706667	0,067000

Table 126 Test membership degree forecast values for Scenario 2 applied to seasonality time series

6.4.4.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
2,289051	1,195455	1,516626	12,503896
2,022416	0,641749	1,119585	2,408998
0,695178	3,779527	1,884285	37,896157

Table 127 Error of type 2(membership degree forecast) values for Scenario 2 applied to seasonality time series

6.4.4.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	8,2423	11,746	15,29	Test granule 1	8,2456	11,747	15,285
Test granule 2	8,2423	11,746	15,29	Test granule 2	1,8856	5,2795	9,0665
Test granule 3	8,2413	11,745	15,288	Test granule 3	-1,3193	1,8658	5,5347
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	2,0583	5,5888	9,5148	Test granule 1	1,6117	4,1985	7,1527
Test granule 2	-1,3126	2,6889	7,3132	Test granule 2	-3,8267	-0,516	3,7362
Test granule 3	-2,7629	0,9784	5,4073	Test granule 3	-5,6128	-2,3761	1,8853

Table 128 Test fuzzy triangles with forecasting method 1 for Scenario 2 applied to seasonality time series

6.4.4.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
4,374534	4,372574	2,364954	1,899576
4,739232	2,620127	1,779553	0,743492
0,059963	0,828939	0,898095	1,171689

Table 129 Error of type 3 for method 1 for Scenario 2 applied to seasonality time series

6.4.4.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	8,2581	11,765	15,307	Test granule 1	8,2615	11,768	15,309
Test granule 2	8,2581	11,765	15,307	Test granule 2	8,2615	11,768	15,309
Test granule 3	8,2581	11,765	15,307	Test granule 3	-4,1897	-1,3886	2,6593
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-4,141	-1,4411	2,4336	Test granule 1	-5,877	-2,231	2,197
Test granule 2	-4,141	-1,4411	2,4336	Test granule 2	-2,268	0,6331	4,957
Test granule 3	-4,141	-1,4411	2,4336	Test granule 3	-6,243	-3,326	0,137

Table 130 Test fuzzy triangles with forecasting method 2 for Scenario 2 applied to seasonality time series

6.4.4.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
4,380277	4,381208	0,114357	0,191907
4,745	4,745935	0,460078	1,180635
0,059463	1,08211	1,088565	1,265316

Table 131 Error of type 3 for method 2 for Scenario 2 applied to seasonality time series

6.4.5. Scenario 3– Optimization with genetic algorithm

6.4.5.i First-phase granules clusterization

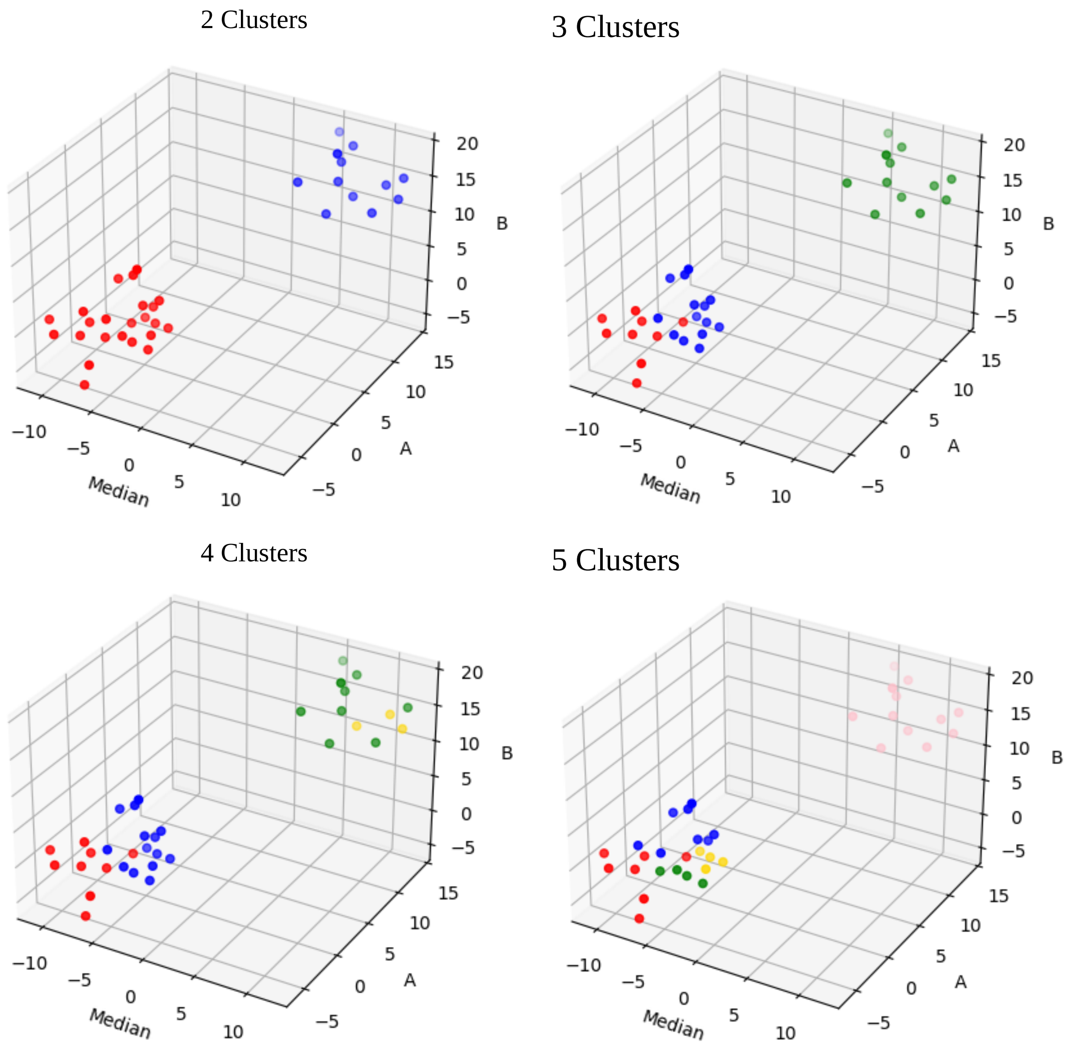


Figure 47 First-phase granules of seasonality time series by applying Scenario 3

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.5.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	13,721568
Cluster 1	17,156402	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	-0,308847	-0,673071
Cluster 1	-0,243971	0	-0,194279
Cluster 2	-0,691918	-0,05265	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	-0,431237	-1,174061	-2,33823
Cluster 1	-1,031113	0	-1,575388	-0,726367
Cluster 2	-2,406757	-0,019115	0	-1,122273
Cluster 3	0,770082	-1,130814	-2,16822	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	-1,929754	-1,058348	-0,892024	-2,92825
Cluster 1	-1,489087	0	-1,428042	-1,87325	-1,032083
Cluster 2	-1,040467	-0,919389	0	-2,879857	-0,437831
Cluster 3	-1,444068	-2,23258	-1,543964	0	0,28484
Cluster 4	-2,205372	-0,763532	-0,831629	-2,339752	0

Table 132 Fuzzy cognitive map weights for Scenario 3 of seasonality time series

6.4.5.iii Test membership values

Cluster 0	Cluster 1
0,920660	0,079340
0,975147	0,024853
0,049448	0,950552

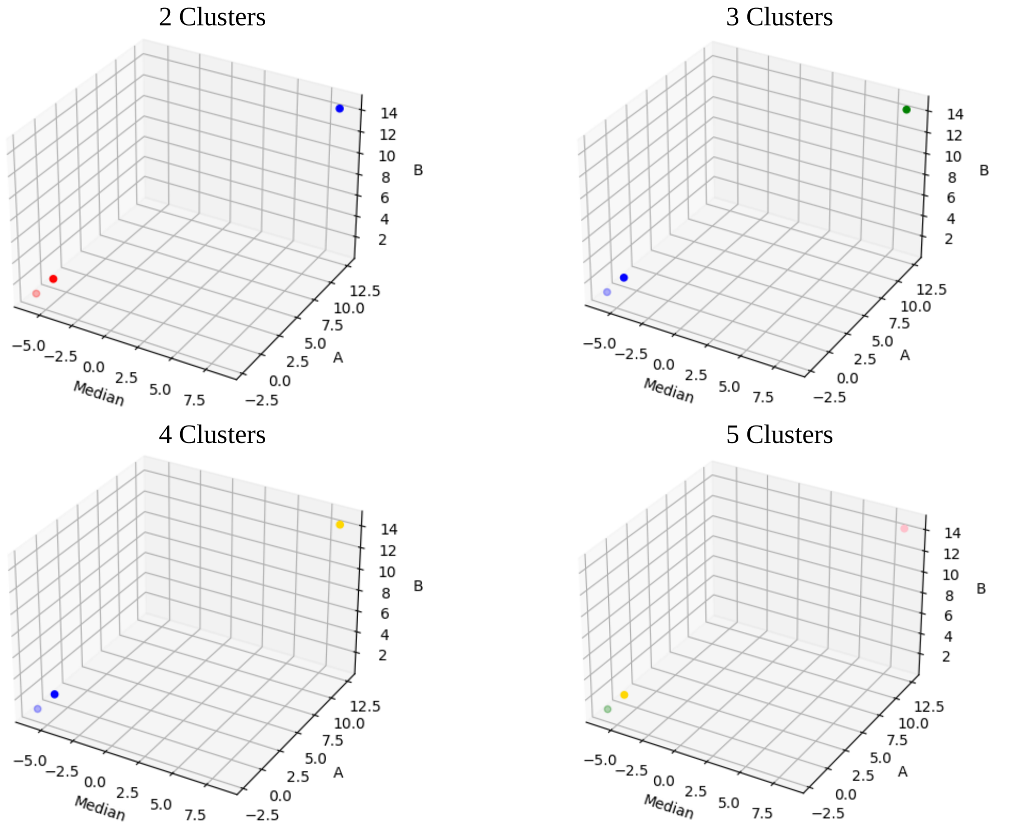
Cluster 0	Cluster 1	Cluster 2
0,421262	0,478126	0,100613
0,452223	0,461968	0,085809
0,232886	0,279536	0,487578

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,304368	0,324851	0,184760	0,186022
0,315850	0,319912	0,181290	0,182947
0,169382	0,197640	0,310918	0,322060

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,214873	0,224633	0,225371	0,235901	0,099223
0,225535	0,215109	0,235073	0,232764	0,091520
0,178943	0,196707	0,184596	0,192929	0,246825

Table 133 Membership degree between first and second-phase granules (clusters) for Scenario 3 of seasonality time series

6.4.5.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 48 Test first-phase granules values and 3D representation of Scenario 3 applied to seasonality time series

6.4.5.v Test membership forecast

Cluster 0	Cluster 1
0,999996	0,835559
0,999990	1,000000
0,999999	1,000000

Cluster 0	Cluster 1	Cluster 2
0,399309	0,462507	0,455207
0,389544	0,453669	0,425401
0,394977	0,455696	0,427167

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,235009	0,290637	0,315150	0,317848
0,224690	0,274945	0,283356	0,303435
0,238532	0,289363	0,291819	0,320329

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,187822	0,241278	0,262235	0,285323	0,239635
0,154547	0,192170	0,206876	0,241124	0,184808
0,206519	0,237219	0,247349	0,285166	0,227272

Table 134 Error of type 2(membership degree forecast) values for Scenario 3 applied to seasonality time series

6.4.5.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,578161	0,126463	0,040361	0,024526
0,951529	0,119320	0,035267	0,015133
0,905991	0,060955	0,013563	0,015230

Table 135 Error of type 2(membership degree forecast) values for Scenario 3 applied to seasonality time series

6.4.5.vii Test fuzzy triangle forecasted values (Method 1)

2 clusters

	a	m	b
Test granule 1	0,7291	3,8757	7,4213
Test granule 2	1,3469	4,5229	8,0682
Test granule 3	1,3469	4,5228	8,0682

3 clusters

	a	m	b
Test granule 1	-0,8341	2,2941	5,8416
Test granule 2	-0,9744	2,1472	5,6937
Test granule 3	-0,9946	2,1275	5,6761

4 clusters

	a	m	b
Test granule 1	2,0276	5,2728	8,7967
Test granule 2	1,9651	5,1786	8,6717
Test granule 3	1,929	5,1295	8,6109

5 clusters

	a	m	b
Test granule 1	-2,6998	0,4391	4,2851
Test granule 2	-2,8048	0,3286	4,1592
Test granule 3	-2,8686	0,294	4,169

Table 136 Test fuzzy triangles with forecasting method 1 for Scenario 3 applied to seasonality time series

6.4.5.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,796685	1,272905	2,248016	0,687849
2,362806	1,575334	2,571871	0,998107
0,607935	0,808467	0,559229	0,95515

Table 137 Error of type 3 for method 1 for Scenario 3 applied to seasonality time series

6.4.5.ix Test fuzzy triangle forecasted values (Method 2)

2 clusters

	a	m	b
Test granule 1	-5,5589	-2,6938	0,9029
Test granule 2	8,2581	11,765	15,307
Test granule 3	8,2581	11,765	15,307

3 clusters

	a	m	b
Test granule 1	-4,1497	-1,53	1,8356
Test granule 2	-4,1497	-1,53	1,8356
Test granule 3	-4,1497	-1,53	1,8356

4 clusters

	a	m	b
Test granule 1	10,366	12,2	13,581
Test granule 2	10,366	12,2	13,581
Test granule 3	10,366	12,2	13,581

5 clusters

	a	m	b
Test granule 1	-3,144	-1,121	0,305
Test granule 2	-3,144	-1,121	0,305
Test granule 3	-3,144	-1,121	0,305

Table 138 Test fuzzy triangles with forecasting method 2 for Scenario 3 applied to seasonality time series

6.4.5.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,351419	0,166818	4,470843	0,493348
4,745	0,382617	4,835969	0,491091
0,059463	1,108132	0,080309	1,111392

Table 139 Error of type 3 for method 2 for Scenario 3 applied to seasonality time series

6.4.6. Scenario 4– Optimization with IPOPT algorithm

6.4.6.i First-phase granules clusterization

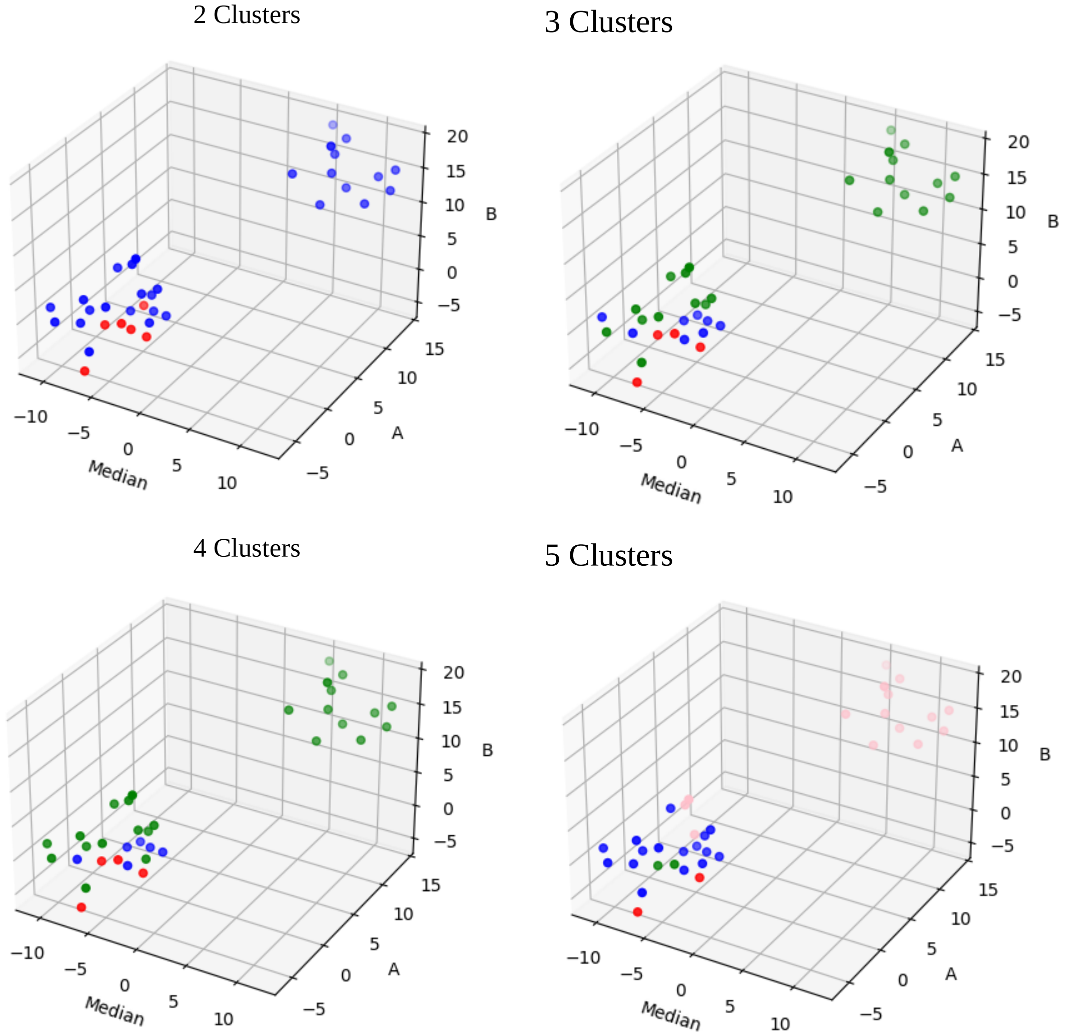


Figure 49 First-phase granules of seasonality time series by applying Scenario 4

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.6.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	5,182037
Cluster 1	17,303744	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	2,528548	2,528548
Cluster 1	1,029242	0	81,597484
Cluster 2	1,109482	4,432173	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,001203	0,001203	0,001203
Cluster 1	119,857316	0	1,502658	1,502658
Cluster 2	113,664487	0,907526	0	0,000182
Cluster 3	2,356658	2,356658	2,356658	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,618682	0,618682	0,618682	0,618682
Cluster 1	0,168388	0	0,385232	0,602242	0,346797
Cluster 2	0,865693	0,865693	0	0,865693	0,865693
Cluster 3	0,511594	0,511594	0,511594	0	0,511594
Cluster 4	0,501195	0,556444	5,535624	0,487997	0

Table 140 Fuzzy cognitive map weights for Scenario 4 of seasonality time series

6.4.6.iii Test membership values

Cluster 0	Cluster 1
0,258993	0,741007
0,499816	0,500184
0,055690	0,944310

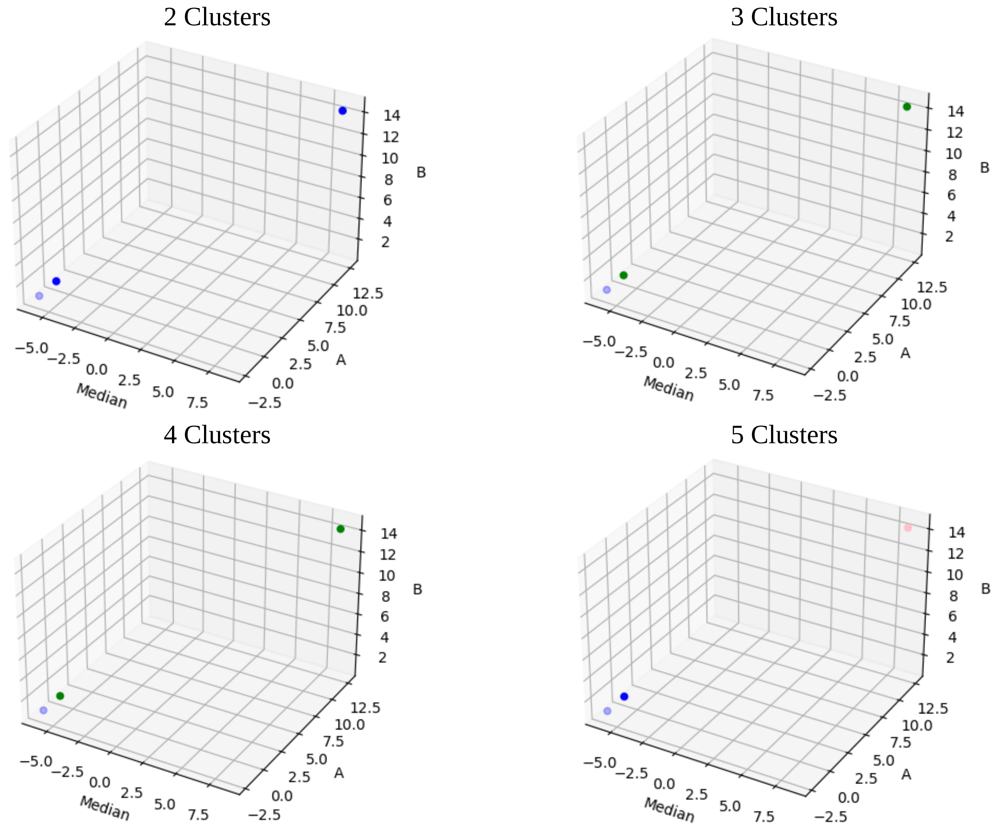
Cluster 0	Cluster 1	Cluster 2
-0,014897	0,404014	0,610883
-0,036118	0,518059	0,518059
-0,003715	0,100763	0,902952

Cluster 0	Cluster 1	Cluster 2	Cluster 3
-0,003954	0,234479	0,384738	0,384738
-0,009892	0,336631	0,336631	0,336631
-0,000851	0,050472	0,490813	0,459566

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,060933	0,477314	-0,191161	0,175600	0,477314
0,164406	0,450457	-0,515777	0,450457	0,450457
0,014939	0,295337	-0,046867	0,043052	0,693539

Table 141 Membership degree between first and second-phase granules (clusters) for Scenario 4 of seasonality time series

6.4.6.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 50 Test first-phase granules values and 3D representation of Scenario 4 applied to seasonality time series

6.4.6.v Test membership forecast

Cluster 0	Cluster 1
0,992415	0,736641
0,978486	1,000000
0,994415	1,000000

Cluster 0	Cluster 1	Cluster 2
0,926800	1,000000	0,615598
0,983457	1,000000	0,995766
0,993609	1,000000	0,996023

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,500301	0,789146	0,486835	0,775369
0,500617	1,000000	1,000000	0,985022
0,500898	1,000000	1,000000	0,997249

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,647711	0,561252	0,712640	0,619753	0,482547
0,813074	0,715894	0,880886	0,773809	0,992492
0,889011	0,783549	0,945458	0,850766	0,997670

Table 142 Test membership degree forecast values for Scenario 4 applied to seasonality time series

6.4.6.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,537927	1,242014	0,724946	1,365507
0,478941	1,500004	1,561149	2,840253
0,884305	1,811944	1,701730	2,731959

Table 143 Error of type 2(membership degree forecast) values for Scenario 4 applied to seasonality time series

6.4.6.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	0,3268	3,4543	7	Test granule 1	-2,3254	0,7527	4,3194
Test granule 2	1,4218	4,6014	8,1467	Test granule 2	-1,0715	2,0647	5,6405
Test granule 3	1,3662	4,5431	8,0884	Test granule 3	-1,0929	2,0439	5,6219
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	1,6143	4,6067	7,8644	Test granule 1	-3,4638	-0,308	3,6049
Test granule 2	2,5207	5,7545	9,2222	Test granule 2	-2,3711	0,8252	4,7172
Test granule 3	2,5466	5,7752	9,2362	Test granule 3	-2,572	0,6238	4,5276

Table 144 Test fuzzy triangles with forecasting method 1 for Scenario 4 applied to seasonality time series

6.4.6.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,658659	0,767927	2,024989	0,444911
2,38863	1,54941	2,759176	1,163842
0,606253	0,815108	0,506104	0,927443

Table 145 Error of type 3 for method 1 for Scenario 4 applied to seasonality time series

6.4.6.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-4,9519	-3,1286	-1,1632	Test granule 1	-4,9195	-2,2611	0,2568
Test granule 2	0,2869	3,6732	7,6266	Test granule 2	-4,9195	-2,2611	0,2568
Test granule 3	0,2869	3,6732	7,6266	Test granule 3	-4,9195	-2,2611	0,2568
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	-4,3589	-1,8983	0,1996	Test granule 1	-5,418	-3,404	-0,658
Test granule 2	-4,3589	-1,8983	0,1996	Test granule 2	5,812	9,3768	13,45
Test granule 3	-4,3589	-1,8983	0,1996	Test granule 3	5,812	9,3768	13,45

Table 146 Test fuzzy triangles with forecasting method 2 for Scenario 4 applied to seasonality time series

6.4.6.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,561386	0,304191	0,305965	0,587583
2,100969	0,18539	0,28098	3,999866
0,674073	1,194762	1,170397	0,194408

Table 147 Error of type 3 for method 2 for Scenario 4 applied to seasonality time series

6.4.7. Scenario 5– Optimization with IPOPT algorithm

6.4.7.i First-phase granules clusterization

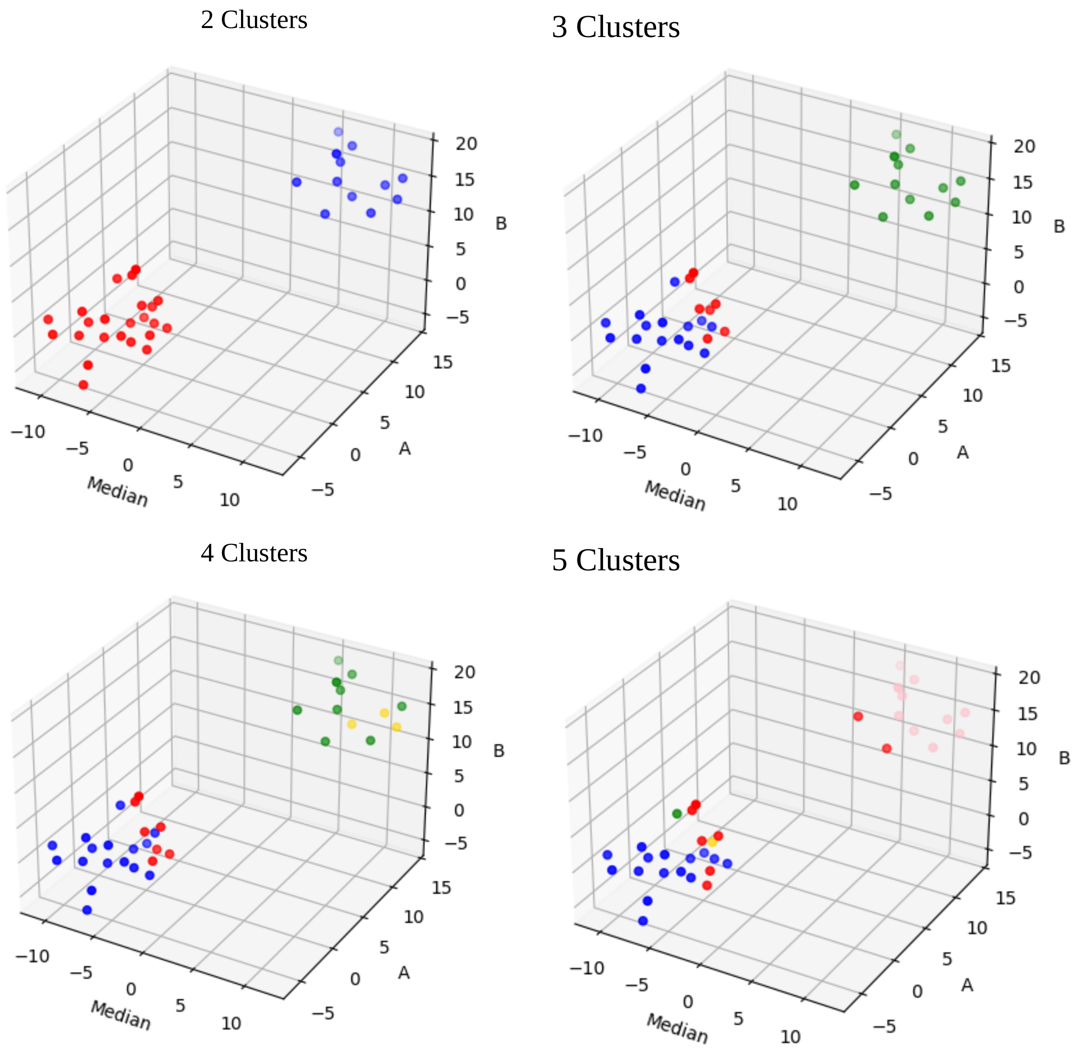


Figure 51 First-phase granules of seasonality time series by applying Scenario 5

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.7.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	0,000013
Cluster 1	0,00001	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	33,148541	1,951136
Cluster 1	5,689041	0	0,058959
Cluster 2	0,000063	0,000203	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	1,941405	0,924147	0,925697
Cluster 1	2,935425	0	0,050614	0,049605
Cluster 2	0,718126	0,756979	0	22,956276
Cluster 3	0,560824	0,69587	1,568819	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	1,445606	25,111041	2,000826	0,613401
Cluster 1	0,488084	0	68,939088	4,688486	1,12367
Cluster 2	1,934404	5,291146	0	1,636871	1,421833
Cluster 3	1,879011	-0,343116	1,517023	0	1,418772
Cluster 4	-51026,11483	1,244921	0,353432	0,559768	0

Table 148 Fuzzy cognitive map weights for Scenario 5 of seasonality time series

6.4.7.iii Test membership values

Cluster 0	Cluster 1
1,071596	-0,071596
1,224016	-0,224016
-0,283927	1,283927

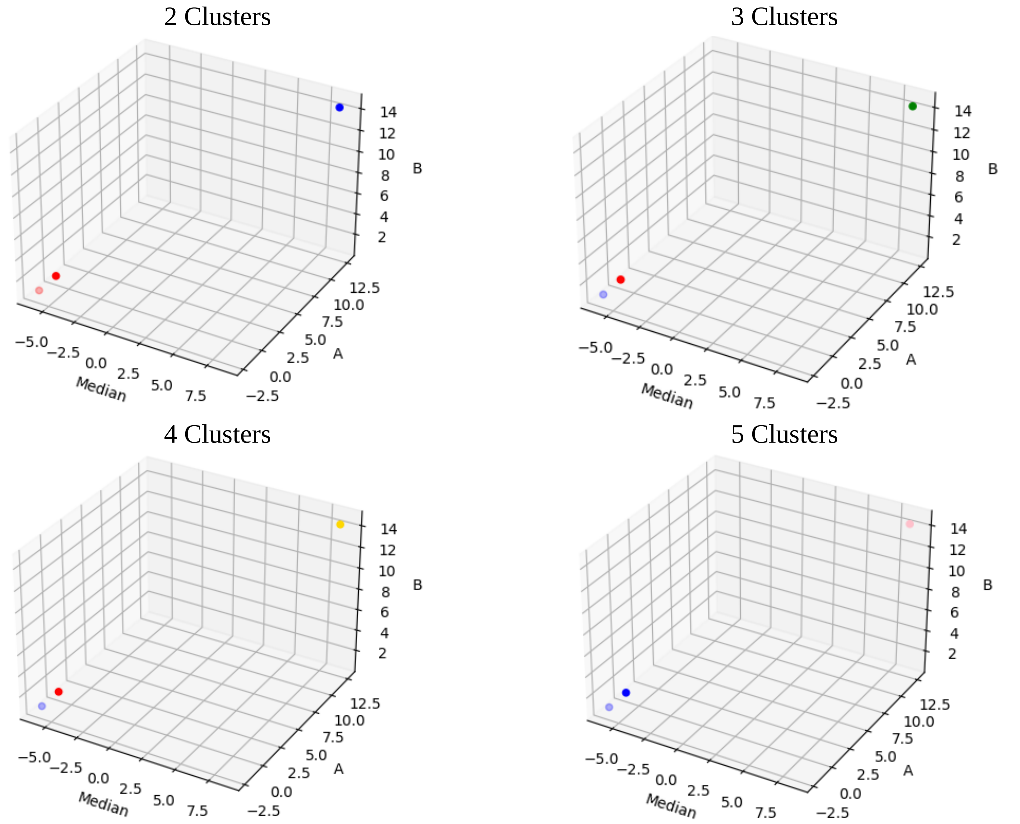
Cluster 0	Cluster 1	Cluster 2
0,640561	0,385815	-0,026376
0,484091	0,576417	-0,060508
-0,670914	-0,208293	1,879207

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,654939	0,398305	-0,026475	-0,026769
0,528928	0,601598	-0,064903	-0,065623
-0,256293	-0,082580	0,644113	0,694760

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
-0,745199	2,877425	-0,532052	-0,628019	0,027845
0,057833	0,834210	0,051270	0,063247	-0,006560
-1,852804	-0,039362	-1,322852	-0,519181	4,734200

Table 149 Membership degree between first and second-phase granules (clusters) for Scenario 5 of seasonality time series

6.4.7.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 52 Test first-phase granules values and 3D representation of Scenario 5 applied to seasonality time series

6.4.7.v Test membership forecast

Cluster 0	Cluster 1
0,500004	0,499999
0,500002	0,500001
0,500002	0,500001

Cluster 0	Cluster 1	Cluster 2
0,004312	0,004941	0,499970
0,757553	0,513499	0,500000
1,000000	0,987121	0,500038

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,751520	0,305125	1,000000	0,699391
0,896965	0,908157	1,000000	0,900484
0,971274	0,938673	1,000000	0,937252

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,000000	0,000000	0,987219	0,655635	1,000000
1,000000	1,000000	0,923792	0,948652	0,671707
1,000000	1,000000	0,999941	0,979970	0,000000

Table 150 Test membership degree forecast values for Scenario 5 applied to seasonality time series

6.4.7.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,653439	0,826917	1,598969	13,735932
1,048399	0,392909	2,296811	2,920448
1,229083	6,123079	2,735338	39,274234

Table 151 Error of type 2(membership degree forecast) values for Scenario 5 applied to seasonality time series

6.4.7.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	1,3468	4,5228	8,0682	Test granule 1	8,0336	11,525	15,061
Test granule 2	1,3469	4,5228	8,0682	Test granule 2	-2,0625	1,0936	4,7658
Test granule 3	1,3469	4,5228	8,0682	Test granule 3	-2,9652	0,1018	3,6932
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	2,4354	6,0006	9,8955	Test granule 1	0,406	2,9462	5,667
Test granule 2	1,4172	4,7129	8,3288	Test granule 2	-3,6516	-0,4016	3,6717
Test granule 3	1,2827	4,5585	8,162	Test granule 3	-5,7005	-2,528	1,5899

Table 152 Test fuzzy triangles with forecasting method 1 for Scenario 5 applied to seasonality time series

6.4.7.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
2,008625	4,301508	2,495732	1,463326
2,362795	1,233508	2,420813	0,768207
0,607935	0,976658	0,606098	1,18674

Table 153 Error of type 3 for method 1 for Scenario 5 applied to seasonality time series

6.4.7.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-5,5589	-2,6938	0,9029	Test granule 1	8,2615	11,768	15,309
Test granule 2	-5,5589	-2,6938	0,9029	Test granule 2	-4,1897	-1,3886	2,6593
Test granule 3	-5,5589	-2,6938	0,9029	Test granule 3	-4,1897	-1,3886	2,6593
	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	7,4388	11,619	16,026	Test granule 1	8,723	12,239	15,63
Test granule 2	7,4388	11,619	16,026	Test granule 2	-6,243	-3,326	0,137
Test granule 3	7,4388	11,619	16,026	Test granule 3	-6,243	-3,326	0,137

Table 154 Test fuzzy triangles with forecasting method 2 for Scenario 5 applied to seasonality time series

6.4.7.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,351419	4,381208	4,352971	4,520177
0,102205	0,485635	4,717572	0,239644
1,206745	1,08211	0,106841	1,265316

Table 155 Error of type 3 for method 2 for Scenario 5 applied to seasonality time series

6.4.8. Scenario 6– Optimization with IPOPT algorithm

6.4.8.i First-phase granules clusterization

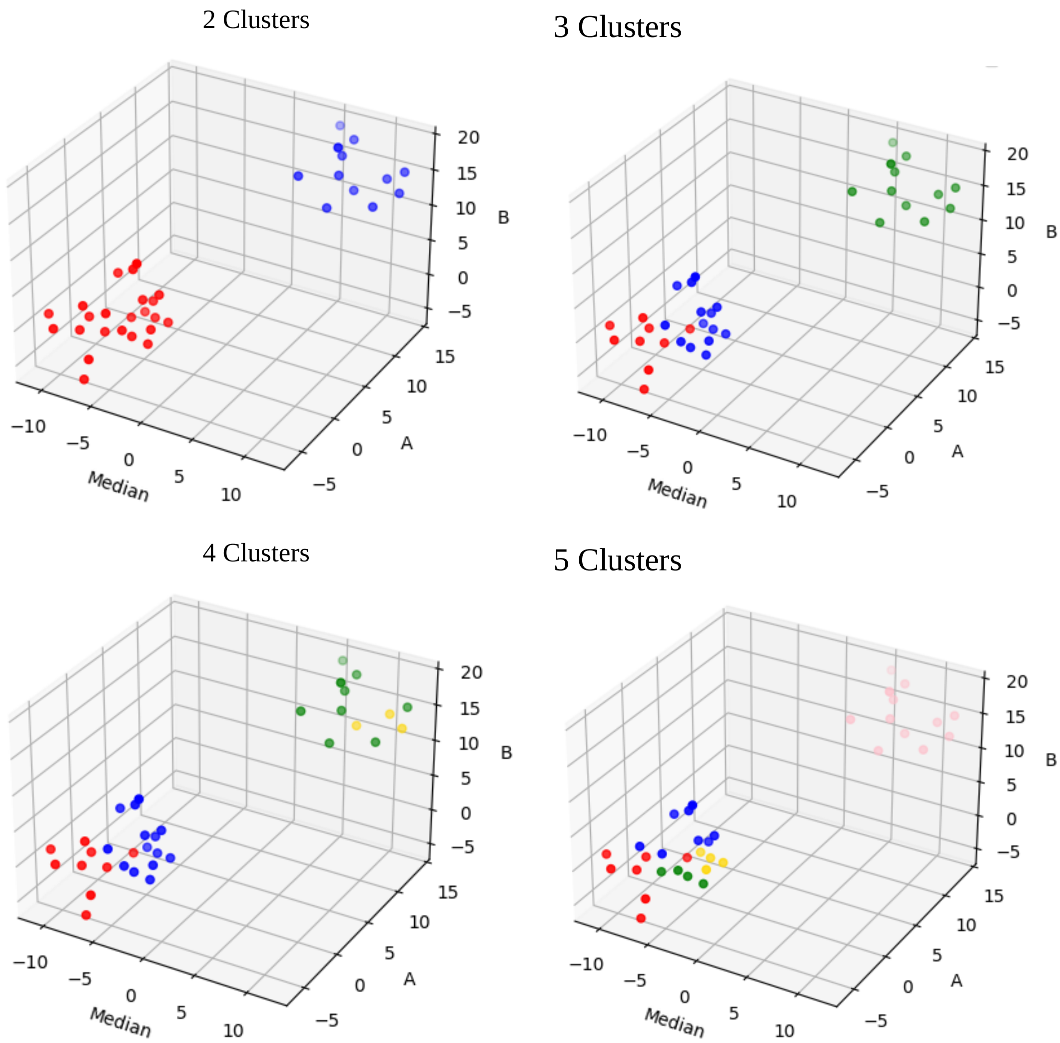


Figure 53 First-phase granules of seasoality time series by applying Scenario 6

Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

6.4.8.ii Fuzzy cognitive map

	Cluster 0	Cluster 1
Cluster 0	0	14,011617
Cluster 1	17,158198	0

	Cluster 0	Cluster 1	Cluster 2
Cluster 0	0	0,00001	0,000033
Cluster 1	0,000006	0	0,000018
Cluster 2	0,000014	0,000011	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3
Cluster 0	0	0,000013	0,000021	0,000021
Cluster 1	0,000009	0	0,000013	0,000013
Cluster 2	0,000023	0,00002	0	0,000013
Cluster 3	0,000105	0,000091	0,00006	0

	Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cluster 0	0	0,000024	0,000023	0,000024	0,000048
Cluster 1	0,000022	0	0,000022	0,000021	0,000036
Cluster 2	0,000041	0,000044	0	0,000039	0,000095
Cluster 3	0,000042	0,000042	0,00004	0	0,00009
Cluster 4	0,000018	0,000017	0,000018	0,000017	0

Table 156 Fuzzy cognitive map weights for Scenario 6 of seasonality time series

6.4.8.iii Test membership values

Cluster 0	Cluster 1
0,920660	0,079340
0,975147	0,024853
0,049448	0,950552

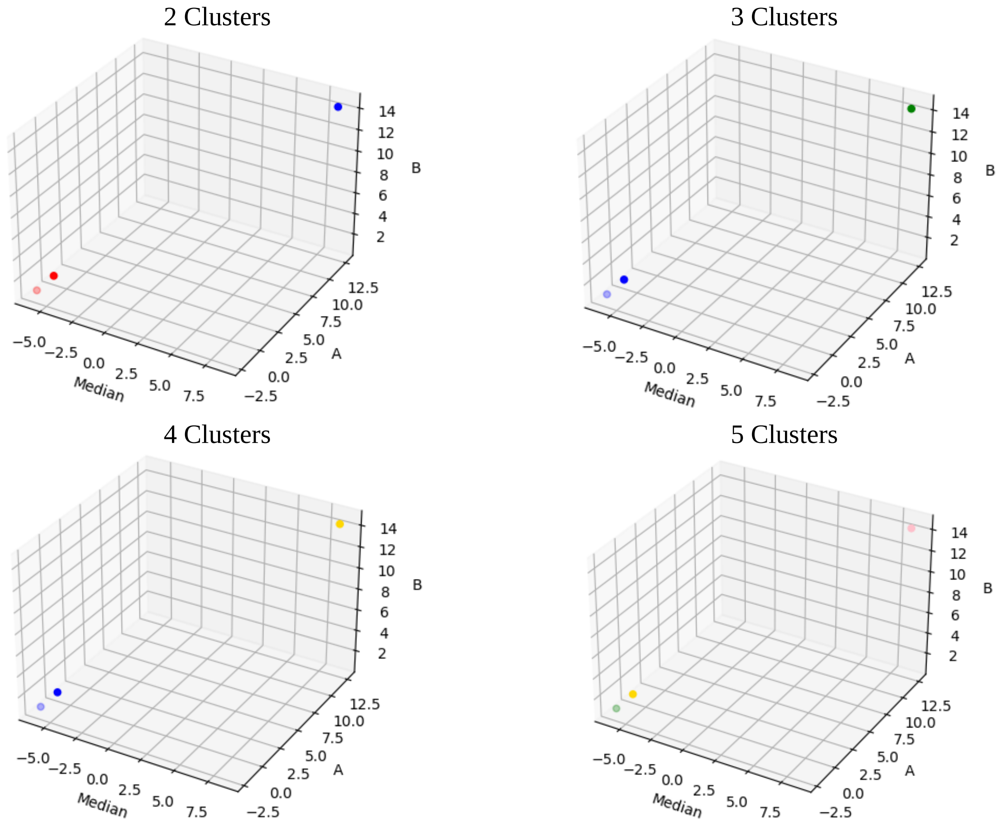
Cluster 0	Cluster 1	Cluster 2
0,421262	0,478126	0,100613
0,452223	0,461968	0,085809
0,232886	0,279536	0,487578

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,304368	0,324851	0,184760	0,186022
0,315850	0,319912	0,181290	0,182947
0,169382	0,197640	0,310918	0,322060

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,214873	0,224633	0,225371	0,235901	0,099223
0,225535	0,215109	0,235073	0,232764	0,091520
0,178943	0,196707	0,184596	0,192929	0,246825

Table 157 Membership degree between first and second-phase granules (clusters) for Scenario 6 of seasonality time series

6.4.8.iv Test membership cluster



Where:

- Cluster 0
- Cluster 1
- Cluster 2
- Cluster 3
- Cluster 4

	a	m	b
Test granule 1	-4,80	-1,80	2,42
Test granule 2	-5,93	-2,20	0,85
Test granule 3	8,64	12,55	14,36

Figure 54 Test first-phase granules values and 3D representation of Scenario 6 applied to seasonality time series

6.4.8.v Test membership forecast

Cluster 0	Cluster 1
0,999997	0,835582
0,999992	1,000000
0,999999	1,000000

Cluster 0	Cluster 1	Cluster 2
0,500005	0,500003	0,500002
0,500005	0,500003	0,500003
0,500005	0,500003	0,500003

Cluster 0	Cluster 1	Cluster 2	Cluster 3
0,500004	0,500002	0,500003	0,500014
0,500007	0,500004	0,500007	0,500032
0,500007	0,500004	0,500007	0,500032

Cluster 0	Cluster 1	Cluster 2	Cluster 3	Cluster 4
0,500006	0,500005	0,500012	0,500011	0,500003
0,500015	0,500013	0,500027	0,500027	0,500009
0,500015	0,500013	0,500027	0,500027	0,500009

Table 158 Test membership degree forecast values for Scenario 6 applied to seasonality time series

6.4.8.vi Error type 2

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,578196	0,166191	0,266921	0,462938
0,951529	0,175287	0,268470	0,465003
0,905992	0,120113	0,268166	0,452989

Table 159 Error of type 2(membership degree forecast) values for Scenario 6 applied to seasonality time series

6.4.8.vii Test fuzzy triangle forecasted values (Method 1)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	0,7292	3,8758	7,4214	Test granule 1	-1,0943	2,0432	5,6227
Test granule 2	1,3469	4,5229	8,0682	Test granule 2	-1,0943	2,0432	5,6227
Test granule 3	1,3469	4,5228	8,0682	Test granule 3	-1,0943	2,0432	5,6227

	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	1,2774	4,5187	8,0836	Test granule 1	-2,884	0,3491	4,3392
Test granule 2	1,2774	4,5188	8,0836	Test granule 2	-2,884	0,3491	4,3391
Test granule 3	1,2774	4,5188	8,0836	Test granule 3	-2,884	0,3491	4,3391

Table 160 Test fuzzy triangles with forecasting method 1 for Scenario 6 applied to seasonality time series

6.4.8.viii Error type 3 (Method 1)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
1,796718	1,191973	2,002184	0,663448
2,362806	1,542494	2,356325	1,011604
0,607935	0,815145	0,60957	0,949249

Table 161 Error of type 3 for method 1 for Scenario applied to seasonality time series

6.4.8.ix Test fuzzy triangle forecasted values (Method 2)

	2 clusters				3 clusters		
	a	m	b		a	m	b
Test granule 1	-5,5589	-2,6938	0,9029	Test granule 1	-7,7189	-4,4659	-0,5168
Test granule 2	8,2581	11,765	15,307	Test granule 2	-7,7189	-4,4659	-0,5168
Test granule 3	8,2581	11,765	15,307	Test granule 3	-7,7189	-4,4659	-0,5168

	4 clusters				5 clusters		
	a	m	b		a	m	b
Test granule 1	10,366	12,2	13,581	Test granule 1	-4,642	-3,123	-1,001
Test granule 2	10,366	12,2	13,581	Test granule 2	-4,642	-3,123	-1,001
Test granule 3	10,366	12,2	13,581	Test granule 3	-4,642	-3,123	-1,001

Table 162 Test fuzzy triangles with forecasting method 2 for Scenario 6 applied to seasonality time series

6.4.8.x Error type 3 (Method 2)

2 Clusters	3 Clusters	4 Clusters	5 Clusters
0,351419	0,944745	4,470843	0,543459
4,745	0,603742	4,835969	0,452339
0,059463	1,357288	0,080309	1,246582

Table 163 Error of type 3 for method 2 for Scenario 6 applied to seasonality time series

6.4.9. Summary tables

6.4.9.i Cluster forecast tables

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	Cluster 0	Cluster 1	Cluster 0
Test granule 2	Cluster 1	Cluster 1	Cluster 1
Test granule 3	Cluster 1	Cluster 1	Cluster 1
3 CLUSTERS			
Test granule 1	Cluster 1	Cluster 1	Cluster 1
Test granule 2	Cluster 0	Cluster 0	Cluster 1
Test granule 3	Cluster 0	Cluster 0	Cluster 2
4 CLUSTERS			
Test granule 1	Cluster 3	Cluster 0	Cluster 3
Test granule 2	Cluster 1	Cluster 0	Cluster 3
Test granule 3	Cluster 1	Cluster 0	Cluster 3
5 CLUSTERS			
Test granule 1	Cluster 2	Cluster 0	Cluster 3
Test granule 2	Cluster 2	Cluster 0	Cluster 3
Test granule 3	Cluster 2	Cluster 0	Cluster 3

Table 164 Second-phase granules forecast accuracy for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	Cluster 0	Cluster 0	Cluster 0
Test granule 2	Cluster 1	Cluster 0	Cluster 1
Test granule 3	Cluster 1	Cluster 0	Cluster 1
3 CLUSTERS			
Test granule 1	Cluster 1	Cluster 2	Cluster 0
Test granule 2	Cluster 1	Cluster 0	Cluster 0
Test granule 3	Cluster 1	Cluster 0	Cluster 0
4 CLUSTERS			
Test granule 1	Cluster 1	Cluster 2	Cluster 3
Test granule 2	Cluster 1/2*	Cluster 2	Cluster 3
Test granule 3	Cluster 1/2*	Cluster 2	Cluster 3
5 CLUSTERS			
Test granule 1	Cluster 2	Cluster 4	Cluster 2
Test granule 2	Cluster 4	Cluster 0/1	Cluster 2/3
Test granule 3	Cluster 4	Cluster 0/1	Cluster 2/3

Table 165 Second-phase granules forecast accuracy for Scenarios optimized by IPOPT algorithm of the seasonality time series

6.4.9.ii Error type 2 tables

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	0,529505	2,289051	0,578161
Test granule 2	0,467887	2,022416	0,951529
Test granule 3	0,875659	0,695178	0,905991
	3 CLUSTERS		
Test granule 1	0,665490	1,195455	0,126463
Test granule 2	1,470650	0,641749	0,119320
Test granule 3	0,891251	3,779527	0,060955
	4 CLUSTERS		
Test granule 1	0,291799	1,516626	0,040361
Test granule 2	0,614654	1,119585	0,035267
Test granule 3	1,327124	1,884285	0,013563
	5 CLUSTERS		
Test granule 1	1,250482	12,503896	0,024526
Test granule 2	2,682495	2,408998	0,015133
Test granule 3	1,944493	37,896157	0,015230

Table 166 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
	2 CLUSTERS		
Test granule 1	0,537927	0,653439	0,578196
Test granule 2	0,478941	1,048399	0,951529
Test granule 3	0,884305	1,229083	0,905992
	3 CLUSTERS		
Test granule 1	1,242014	0,826917	0,166191
Test granule 2	1,500004	0,392909	0,175287
Test granule 3	1,811944	6,123079	0,120113
	4 CLUSTERS		
Test granule 1	0,724946	1,598969	0,266921
Test granule 2	1,561149	2,296811	0,268470
Test granule 3	1,701730	2,735338	0,268166
	5 CLUSTERS		
Test granule 1	1,365507	13,735932	0,462938
Test granule 2	2,840253	2,920448	0,465003
Test granule 3	2,731959	39,274234	0,452989

Table 167 Error of type 2 in forecast accuracy of membership degree functions for Scenarios optimized by IPOPT algorithm of the seasonalitytime series

6.4.9.iii Error type 3 tables (Method 1)

	Scenario 1	Scenario 2	Scenario 3
2 CLUSTERS			
Test granule 1	1,658919401	4,374534368	1,796685144
Test granule 2	2,388210802	4,739231626	2,362806236
Test granule 3	0,606289001	0,059963432	0,607935302
3 CLUSTERS			
Test granule 1	1,021308204	4,374456763	1,272904656
Test granule 2	2,034387528	2,618218263	1,575334076
Test granule 3	1,069066104	0,828939522	0,808466948
4 CLUSTERS			
Test granule 1	2,103359202	2,366064302	2,248015521
Test granule 2	1,241481069	1,778340757	2,571870824
Test granule 3	0,97569339	0,898092827	0,559229255
5 CLUSTERS			
Test granule 1	0,488713969	1,900543237	0,687849224
Test granule 2	1,102394209	0,743151448	0,998106904
Test granule 3	0,927392405	1,171690577	0,955150492

Table 168 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
2 CLUSTERS			
Test granule 1	1,658658537	2,008625277	1,796718404
Test granule 2	2,38863029	2,3627951	2,362806236
Test granule 3	0,606253165	0,607935302	0,607935302
3 CLUSTERS			
Test granule 1	0,767926829	4,301507761	1,191973392
Test granule 2	1,5494098	1,233507795	1,542494432
Test granule 3	0,815108298	0,976658228	0,815144866
4 CLUSTERS			
Test granule 1	2,024988914	2,495731707	2,002184035
Test granule 2	2,759175947	2,420812918	2,356325167
Test granule 3	0,506104079	0,606098453	0,60956962
5 CLUSTERS			
Test granule 1	0,444911308	1,463325942	0,663447894
Test granule 2	1,163841871	0,768207127	1,011603563
Test granule 3	0,927443038	1,186739803	0,949248945

Table 169 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the seasonality time series

6.4.9.iv Error type 3 tables (Method 2)

	Scenario 1	Scenario 2	Scenario 3
	2 CLUSTERS		
Test granule 1	0,561385809	4,380277162	0,351419069
Test granule 2	2,10096882	4,745	4,745
Test granule 3	0,674073136	0,059462729	0,059462729
	3 CLUSTERS		
Test granule 1	0,304190687	4,381208426	0,166818182
Test granule 2	0,511202673	4,745935412	0,382616927
Test granule 3	1,318908579	1,082109705	1,108132208
	4 CLUSTERS		
Test granule 1	4,400110865	0,114356984	4,470842572
Test granule 2	0,280979955	0,460077951	4,83596882
Test granule 3	1,170396624	1,088565401	0,080309423
	5 CLUSTERS		
Test granule 1	0,587583149	0,191906874	0,493348115
Test granule 2	0,359020045	1,180634744	0,491091314
Test granule 3	1,266666667	1,265316456	1,111392405

Table 170 Error of type 3 with forecasting method 1 for values of first-phase granule for Scenarios optimized by genetic algorithm of the seasonality time series

	Scenario 4	Scenario 5	Scenario 6
	2 CLUSTERS		
Test granule 1	0,561385809	0,351419069	0,351419069
Test granule 2	2,10096882	0,1022049	4,745
Test granule 3	0,674073136	1,206745429	0,059462729
	3 CLUSTERS		
Test granule 1	0,304190687	4,381208426	0,944745011
Test granule 2	0,185389755	0,485634744	0,603741648
Test granule 3	1,194762307	1,082109705	1,357288326
	4 CLUSTERS		
Test granule 1	0,305964523	4,352971175	4,470842572
Test granule 2	0,280979955	4,717572383	4,83596882
Test granule 3	1,170396624	0,106841069	0,080309423
	5 CLUSTERS		
Test granule 1	0,587583149	4,520177384	0,54345898
Test granule 2	3,99986637	0,239643653	0,45233853
Test granule 3	0,194407876	1,265316456	1,246582278

Table 171 Error of type 3 with forecasting method 2 for values of first-phase granule for Scenarios optimized by IPOPT algorithm of the seasonality time series

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